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UNIVERSITY OF ALBERTA

# WRITING IN MATHEMATICS:
# TEACHER AWARENESS AND REFLECTION

BY

Elizabeth M. Mowat

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of MASTER OF EDUCATION.

DEPARTMENT OF SECONDARY EDUCATION

Edmonton, Alberta

Fall 1992

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ISBN 0-315-77210-7

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## ABSTRACT

This study explored the ways in which the use of writing activities in the mathematics classroom provided opportunities for teachers to become more aware of the effectiveness of classroom instruction, students' attitudes and feelings, and students' understanding of mathematical content, and to reflect on their own conceptions of mathematics and teaching. Research was carried out at a large, urban senior-high school. Four teachers of Mathematics 30 participated.

The study consisted of two phases. During the first phase, teachers administered writing activities to their classes. Interviews were conducted to determine the insights students' writing conveyed to teachers. During the second phase, teachers began to develop purposeful programs for integrating writing activities into their programs of instruction. Interviews revealed the areas and degrees of teachers' reflection.

All four teachers demonstrated that writing activities did increase their awareness of the effectiveness of classroom instruction. Teachers became more aware of the effectiveness of mathematics instruction in general, different teaching techniques, instruction of specific topics, classroom communication, and instruction on writing.

Teachers also became more aware of students' attitudes and feelings. Responses revealed students' emotional

reactions, attitudes to studying mathematics, and conceptions of mathematics.

Teachers became more aware of students' understanding of mathematics content. Student responses contained information about students' background knowledge, knowledge of specific Mathematics 30 concepts and processes, use of metacognitive strategies, use of mathematical language, and modes of understanding mathematics.

Teachers showed their awareness of this information by specifically using writing activities to elicit such feedback, and by modifying their opinions of students and instruction in response.

The three teachers who participated in both phases of the study revealed their reflections on mathematics and teaching through changes in their use of writing activities. From viewing writing as a means of assessing students' understanding, teachers moved to seeing writing as a mode of learning and as a means of empowering students as thinkers.

All four teachers, with widely-varying conceptions of mathematics and teaching, were able to successfully incorporate writing activities in their mathematics classroom and found it to be a useful instruction technique.

## ACKNOWLEDGEMENTS

I extend thanks to Dr. Sol Sigurdson for his advice and encouragement over the past year.

I am also indebted to the four teachers who participated in this study for their insights, contributions, and enthusiasm.

Finally, I must acknowledge the support given me by all of my family, particularly my husband, Ian, and my daughters, Mary and Katherine.

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# Chapter I - The Problem

## Introduction

In recent years, it has been recognized that there is a need for students to be prepared adequately for the challenges they will face as citizens, consumers, workers, and students at the post-secondary level. In order to accomplish this, the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 1989) suggests that mathematics instruction should "encourage and enable students to value mathematics, gain confidence in their own mathematical ability, become mathematical problem solvers, communicate mathematically and reason mathematically" (p.123).

This view of mathematics learning suggests the need for changes both in instructional strategies and in the roles of student and teacher. Instructional methods that assist students in investigating, constructing meanings, and debating and choosing among alternate strategies in problem-solving are recommended. The student is empowered to learn by doing mathematics. The teacher is encouraged to become not a dispenser of knowledge, but a facilitator of learning, a guide, or coach for the student (NCTM, 1989).

Among the many strategies receiving increased attention

is student communication in the form of writing. "Because writing in mathematics involves many of the thought processes teachers would like to foster in their students, every mathematics teacher must seriously consider the use of writing as a part of the daily routine of the mathematics classroom" (Miller & England, 1989, p. 300). In response, an increasing number of mathematics educators have begun to use various types of writing activities in their mathematics classrooms. Writing is being advocated as a valuable tool in mathematics instruction.

## Need for the Study

As with the incorporation of any new teaching strategy, two questions are central to any discussion of the uses of Writing to Learn activities in mathematics:

1. can writing enhance the quality of students' learning of material encountered in the classroom; and
2. can a teacher use writing to enhance his or her understanding of that learning (Marwine, 1989)?

Material encountered in the mathematics classroom includes not just the mathematical content studied, but perspectives on the field of mathematics itself, and attitudes to the learning of mathematics and to learning in general. Students' learning of such materials is affected

both by what is experienced in the classroom and by the personal attitudes and feelings of the student.

The teacher's attitudes and feelings also affect student learning (Peterson, Carpenter & Fennema, 1989). These views are, in turn, influenced by classroom interactions (Cobb, Yackel & Wood, 1988). As teachers' perceptions are modified by these interactions, so too are the ways in which teachers relate to students. The ways in which teachers present mathematical concepts and organize instruction are also affected.

New instructional activities offer new means for the teacher to interact with his or her students. They also offer new avenues for the teacher to use in obtaining information about students and their learning. Indeed, it is necessary for the teacher to have such information. "Pedagogy, as a form of knowledge, implies that one has a realistic knowledge of children, that one 'understands' children and youths: how young people are, what they think about, how they look at the world, what they do" (Van Manen, 1992, p. 14). The introduction of an instructional activity such as writing creates new opportunities for teachers to refine their perceptions of students and classroom interactions.

Most research on writing, both in mathematics and in

other content areas, has focused on students' learning. Researchers have identified many cognitive and affective benefits that students may derive from writing in the classroom.

Few studies, however, have examined whether writing assists teachers to better understand the interactions between students and the mathematics presented in the classroom (Borasi & Rose, 1989; Miller & England, 1989). As this understanding is an important influence on students' learning, "research on teaching should focus on teachers' thoughts, decisions and behaviours" (Shavelson & Stern, 1981, p. 496). Several researchers have suggested that writing activities may be of even more benefit to teachers than to students (Miller and England, 1989).

## Statement of the Problem

The purpose of this study is to investigate whether the incorporation of writing activities can assist teachers in increasing their awareness of what occurs in the mathematics classroom. The aim of the study is to determine whether a program of writing activities provides opportunities for the teacher:

1. to become more aware of the effectiveness of classroom instruction;

2. to become more aware of students' attitudes and feelings;
3. to become more aware of students' understanding of mathematical content; and
4. to reflect on their own conceptions of mathematics and teaching.

## Significance of the Study

Increased knowledge of these areas should assist the teacher in perceiving more clearly how and what students are learning in the mathematics classroom.

Teacher awareness of the effectiveness of classroom instruction. Student writing offers immediate feedback to the teacher on the success or lack of success of different instructional methodologies. Such success may be measured in terms of efficiency, student mastery of concepts, or appeal to students. Understanding of these factors may lead to changes in the instructional techniques used and, consequently, to improvements in instruction (Azzolino, 1990; Borasi and Rose, 1989; Miller and England, 1989).

Teacher awareness of students' attitudes and feelings. By becoming more aware of what is going on in the minds and hearts of students, the teacher may become more empathetic. The classroom atmosphere may become more caring and

supportive for students (LeGere, 1991). The level of trust and mutual respect between student and teacher may increase as writing activities are used (Keith, 1988).

Teacher awareness of students' understanding of mathematics content. Through student writing, teachers may become more aware of students understanding of mathematical concepts and skills. Writing activities can act as diagnostic tools , identifying students' misconceptions and difficulties, and enabling the teacher to individualize instruction more effectively (Borasi & Rose, 1989; Nahrgang & Petersen, 1986). Immediate changes and improvements in instruction may take place.

Teacher awareness of their own conceptions of mathematics and teaching. Teachers may become more aware of their own conceptions of teaching and mathematics. The pressure of time and work provide few opportunities for classroom teachers to reflect on these. But dealing with the conflicts between previously established forms of teaching and newly introduced instructional techniques can produce changes in teachers' beliefs about learning, teaching, and mathematics itself (Wood, Cobb & Yackel, 1991). Such reflections may lead to long-term changes in how a teacher approaches mathematics instruction. Changes may also occur in the way in which a teacher views relations with students.

## Delimitations of the Study

1. This study explores the implementation of writing activities in the Mathematics 30 course as outlined by Alberta Education (1991a).
2. Students taking this course are in the final year of an academic stream of the mathematics program.
3. The study examines the perceptions of four mathematics teachers at a large urban senior high school. The school draws students from both moderate and high socio-economic areas.
4. Teachers' perceptions as expressed in informal interviews and as interpreted by the researcher make up the main sources of data for the study.

## Limitations of the Study

Every human endeavor is affected by basic assumptions and beliefs. Thus, the researcher's views have certainly influenced not only what was done in the study, but what its findings were. Three areas would seem to be possible sources of bias.

1. Personal background. This study was conducted while the researcher was on leave from her school district. The researcher taught mathematics for nineteen years, of which the last thirteen were spent at a large, urban

high school. The researcher had taught Mathematics 30 many times, but not in its current form.

2. Knowledge of participants. The researcher personally knew and taught with all participating teachers. The researcher has great respect for the personal and professional qualities of the participants. They were considered colleagues, rather than subjects in the study.
3. View of writing. Reading about writing in content areas caused the researcher to form her own perspective on the purposes and aims of writing activities in mathematics. These certainly influenced the design of the study, what was said to teachers throughout, and the ways in which the findings were interpreted.
4. View of learning. The researcher views learning as the process of constructing understanding for one's self.
5. View of mathematics. The researcher views mathematics as "a fallible social construct ... a process of inquiry and coming to know, a continually expanding field of human creation, not a finished product" (Ernest, 1991, p. xii).

Definition of Terms

1. Writing. Writing can be defined as a process learned initially only with formal systematic instruction, "... originating and creating a unique verbal construct that is graphically recorded" (Emig, 1977, p. 123). A wide variety of symbols and iconic representations such as the numbers, variables, equations, and graphs used in mathematics and science meet these criteria.
2. Writing activity. A writing activity is an educational procedure designed to stimulate students' learning through the act of writing on the part of the student.
3. Writing prompt. A writing prompt is a brief written cue intended to urge students to write and to direct their writing. It might take the form of a phrase needing completion. It might ask a question that students are to answer. It might present a situation that students are to describe in their own words. Writing prompts were classified into four categories:
   a. content writing activities, which were primarily intended to elicit feedback on students' understanding of mathematical concepts, skills, and processes;
   b. affective writing activities, which were primarily intended to elicit feedback on students' attitudes

and feelings;

c. instructional writing activities, which were primarily intended to elicit feedback on the effectiveness of classroom instruction;

d. creative writing activities, which were primarily intended to involve the student in mathematics personally and imaginatively.

4. Student response. A student response is writing done by the student in reply or in reaction to a writing prompt.

5. Teacher comment. A teacher comment is an observation or remark expressing a teacher's perception, opinion or attitude.

## Organization of the Study

Chapter II reviews the literature concerning writing in content areas and, particularly, writing in mathematics. This chapter is divided into three sections. The first examines the literature on writing and learning, in general. Next, different paradigmatic approaches for incorporating writing in content areas are described. Finally, research into the use of writing activities in mathematics is examined.

Chapter III outlines the design of the study. The setting and procedures of the study are described, as are the

methods of data collection and analysis.

Chapter IV reports findings related to the first three questions asked by the study: Did a program of writing activities provide opportunities for teachers to become more aware of:

1. the effectiveness of classroom instruction;
2. students' attitudes and feelings; and
3. students' understandings of mathematical content?

Data from students' written responses and teachers' comments made in interviews and questionnaires are presented and analyzed.

Chapter V examines the fourth question asked in the study: Does a program of writing activities provide opportunities for teachers to reflect on their own conceptions of mathematics and teaching? Data from teachers' comments made in interviews and questionnaires will be related to teachers' implementation of writing activities in their classrooms.

Chapter VI restates the basic questions of the study, and briefly summarizes the research as carried out. Findings are reviewed and discussed. Finally, suggestions for further research are given.

# Chapter II - Review of the Literature

## Introduction

> Words strain,
> Crack and sometimes break, under the burden
> Under the tension, slip, slide, perish,
> Decay with imprecision, will not stay in place,
> Will not stay still. (Elliot, 1936, p. 121)

At first glance, it is not clear that writing has anything to offer the teacher of mathematics. Words are 'slippery'. A piece of writing has multiple interpretations, as the 'horizons of meaning' of the reader and the writer of the text mesh and fuse (Hekman, 1984). In contrast, the numbers, symbols, and terms of mathematics appear to 'stay still'. But mathematics too is changeable and open to different interpretations. Mathematics is full of human values: mathematicians are fallible. Communication and language play important parts in establishing and verifying the 'truths' of mathematics (Ernest, 1991). Therefore, communication and language have a proper place in mathematics education.

The purpose of the study is to explore some of the roles of written language in the mathematics classroom. It will examine whether writing activities can provide opportunities for teachers:

1. to become more aware of the effectiveness of classroom instruction;
2. to become more aware of students' attitudes and feelings;
3. to become more aware of students' understanding of mathematical content; and
4. to reflect on their own conceptions of mathematics and teaching.

This chapter contains three sections. First , it discusses the unique characteristics of writing. Next, it reviews three perspectives from which writing in content areas can be viewed and gives an overview of research in writing in content areas. Lastly, research into writing in mathematics is examined.

## Writing

Many cognitive functions require verbalization in order to develop fully (Vygotsky, 1962; Bruner, 1971; Luria & Yudovich, 1971). Vygotsky (1962) declares that "thought is born through words. ... thought unembodied in words remains a shadow" (p. 72). 'Inner speech' is the link between thought and language. By talking to themselves, by carrying on inner conversations, people are able to question, consider, evaluate, and reason. To understand, people must manipulate information gained from sensory experience and articulate it,

internally or perhaps externally (Fulwiler, 1987).

Thoughts can be expressed externally by talking or by writing. As mentioned previously, writing can be defined as "... originating and creating a unique verbal construct that is graphically recorded" (Emig, 1977, p. 123). Other types of verbal communication - talking, listening and reading - do not meet these criteria. While talking certainly involves originating and creating a unique verbal construct, it is not graphically recorded.

Writing is more than talk recorded. Linguists and psychologists suggest that writing and talking emanate from different organic sources (Emig, 1977). Moreover, "written speech is a separate linguistic function, differing from oral speech in both structure and mode of functioning" (Vygotsky, 1962, p. 98).

Writing has a number of characteristics that oral speech does not share. Emig (1977) has categorized many of these differences.

1. Writing is a learned behaviour; talking is a natural, even irrepressible behaviour.
2. Writing is an artificial process; talking is not.
3. Writing is a technological device - not the wheel, but early enough to qualify as primary technology; talking is organic, natural, earlier.
4. Most writing is slower than most talking.
5. Writing is stark, barren, even naked as a medium; talking is rich, luxuriant, inherently redundant.
6. Talk leans on the environment; writing must provide its own context.
7. With writing, the audience is usually absent; with

talking the listener is usually present.

8. Writing usually results in a visible graphic product; talking usually does not.
9. Perhaps because there is a product involved, writing tends to be a more responsible and committed act than talking.
10. It can even be said that throughout history, an aura, an ambience, a mystique has usually encircled the written word; the spoken word has for the most part proved ephemeral and been treated mundanely.
11. Because writing is often our representation of the world made visible, embodying both process and product, writing is more readily a form and source of learning than talking. (pp. 123-124)

Other distinctions between writing and talking have been identified. First, writing gives one power to reflect on the verbalization itself. Talking does not (Ong, 1982). "Writing virtually forces a remoteness of reference of the language user" (Bruner, 1971, p. 47). Writing puts thoughts outside the writer. "Writing, by objectifying words, and by making their meanings available for much more prolonged and intensive scrutiny than is possible orally, encourages private thought" (Goody & Watt, 1963, p. 346).

The writer is able to, even encouraged to, read his own writing, repeatedly reviewing , analyzing, and synthesizing. This recursive process transforms a "sequential chain of connections (into) a simultaneous, self-reviewing structure" (Luria & Yudovich, 1971, p. 118).

One can easily review what one writes, but one cannot review what one says.

> I think there is an enormous difference between speaking and writing. One can reread what one rewrites. But one can read slowly or quickly; in other words, you do not know how long you will have to deliberate over a sentence. ... If I listen to a tape recorder, the listening speed is determined by the speed at which the tape turns and not by my own needs. Therefore, I will always be lagging behind or running ahead of the machine. (Sartre, 1975, p. 23)

Second, writing represents a multi-representational, integrative form of language use. Jerome Bruner (1971) describes three modes of learning:

1. learning by doing, that is, learning with the hand, or enactive learning;
2. learning by depiction in an image, that is, learning with the eye, or iconic learning; and
3. learning by restating, that is, learning with the brain, or symbolic learning.

Writing involves all three of these modes of learning. Talking uses the brain, but not the hand or the eye.

## Writing in Content Areas

In this section, three perspectives on writing in content areas will be discussed. Research in the area will be described briefly.

The incorporation of writing activities in subject disciplines began with what has been called 'writing across the curriculum' . Programs of 'writing across the curriculum' originated at colleges and universities in

response to a perceived need to improve student writing and thinking ability. In 1974, Carleton College in Northfield, Minnesota started the first program calling for college wide responsibility for writing instruction. The movement to establish such programs grew until, in 1986, 36% of all colleges and universities reported having 'writing across the curriculum' programs (Fulwiler & Young, 1990).

Several colleges sponsored training programs for secondary and elementary teachers. It was felt that the idea of writing in content areas should 'be planted' before students get to post-secondary institutions (Fulwiler & Young, 1990). From such beginnings, the use of writing in content areas filtered out into the public school system. Writing has been made a part of instruction in disciplines from English to Mathematics, from Psychology to Business Education, from Chemistry to Home Economics.

The term 'writing in content areas' describes a wide variety of writing and writing-related activities. It includes: freewriting, focused writing, in-class writing, out-of-class writing, structured writing, unstructured journal writing, writing for an audience, writing for one's own self, writing that is evaluated, writing that is not, poetry, and technical reports.

Writing activities can be classified as expressive, poetic, or transactional. These terms were first used in a

study of British schools by Britton, Burgess, Martin, McLeod and Rosen (1975). They categorized writing according to its function from the point of view of the writer. Transactional writing is used by the writer "to communicate to an audience, to inform, persuade or instruct them" (p. 160). In contrast, the purpose of expressive writing is for "the writer to explore what he or she thinks, feels or knows" (p. 90). Britton's third category is poetic writing, which he defines as "language used as art" (p. 161). These types of writing are not disjoint, as expressive writing can naturally develop into transactional or poetic writing.

Programs of writing in content areas appear to fall into three separate categories, each determined by a different perspective on learning, teaching, and writing. Kroll (1980) suggests that these paradigms may be based on different views of human development, leading to opposing pedagogies and distinct definitions of the main tasks of education. Knoblauch and Brannon (1983) suggest that a basic view of knowledge itself is involved. Each of these three paradigms, 'learning to write', 'writing to learn', and 'writing to think', is discussed below.

Learning to write. This phrase describes the perspective of the original 'writing across the curriculum' programs, what Connolly (1986) calls 'grammar across the curriculum'. This paradigm is based on a view of knowledge

as a "stable and bounded artifact, a collection of information, a set of facts and ideas to be delivered to students" (Knoblauch & Brannon, 1983, p. 467). Knowledge is discovered raw in nature by individual intellects (Connolly, 1986). As a consequence, the goal of instruction is to 'cover' a subject. The teacher is an expert who passes knowledge on to students who passively record and store it. Learning is receiving information. Knowing is retaining information. Kroll (1980) comments that this paradigm implicitly views education as the essential source of development in children.

The role of writing in education is to display clearly or to test the student's memory and mastery of facts and skills (Knoblauch & Brannon, 1983). Teachers emphasize "... the composed product, rather than the composing process; the analysis of discourse into words, sentences, and paragraphs; the classification of discourse into description, narration, exposition and argumentation; the strong concern with usage (syntax, spelling, punctuation) and with style (economy, clarity, emphasis); the preoccupation with the informal essay and the research paper; and so on" (Young, 1978, p. 31).

In a content area, students are encouraged to write in order to master the formal conventions of writing for that discipline, but, most importantly, to improve the quality of their writing in general. While writing becomes the concern

of all educators, the English department largely maintains control of how writing is taught and evaluated.

Writing to learn. This view of writing requires a different perspective on education as a whole. Knowledge is viewed as the making of connections, not as an inert body of information. Learning is constructing knowledge for oneself, making meaning from the materials of his or her experience (Knoblauch & Brannon, 1983). Knowing is the consciousness of "... patterns of relationships endlessly evolving through accretion, disintegration and reconstruction" (Knoblauch & Brannon, 1983, p. 467). Kroll (1980) notes that this paradigm acknowledges that "the human organism contains the seeds of its own growth" (p. 479). The student produces, applies, and extends his or her knowledge in the same way a mathematician or scientist does. The teacher moves from dispensing information, to stimulating conceptual involvement and investigation (Knoblauch & Brannon, 1983). 'Writing to learn' strategies place the student at the center of the teaching-learning process (Stevens, 1985).

With this shift in the concept of knowledge comes a shift in the concept of writing and composition (Davis, 1984; Knoblauch & Brannon, 1983; Connolly, 1986). Writing is viewed not as a product, but as a recursive process (Forman & Katsky, 1986).

> Process-oriented approaches are not simply an alternative way to achieve subject-area goals. Instead, when these approaches are implemented most effectively, they bring with them a fundamental shift in the nature of teaching and learning. (Langer & Applebee, 1987, p. 67)

Janet Emig believes writing "represents a unique mode or learning - not merely valuable, not merely special, but unique" (Emig, 1977, p. 122). It simultaneously involves the eye, hand and brain. Both right and left brain activity are present. The three tenses of experience - past, present, and future - are connected through analysis and synthesis forcing the writer to be clear about relationships. The product is available for immediate feedback to the writer.

Writing enables the formation of meaning, the construction of knowledge. In both writing and learning students construct meaning, actively building connections between what is being learned and what is known (Mayer, Lester & Pradl, 1983; Borasi & Rose, 1989).

> While musicians may speak to one another through their notes, or mathematicians may spring to a blackboard to talk in numbers, natural language remains the most important mediator of concepts we do not yet fully hold. (Connolly, 1986, p. 4)

The natural language of speech and writing acts as a "metadiscourse" for all symbol systems, be these musical, mathematical, graphic, kinetic, or verbal.

Writing forces the learner to engage in the "deliberate structuring of the web of meaning" (Vygotsky, 1962, p. 100). The student discovers new ideas that would not have been thought of otherwise (Connolly, 1986).

> Composing consists of joining bits of information into relationships, many of which have never existed until the composer utters them. Simply by writing - that is, by composing information - you become aware of the connections you make, and you thereby know more than you knew before starting to write. In its broadest sense, knowledge is an awareness of relationships among pieces of information. As you compose, your new knowledge is your awareness of those relationships. (Van Nostrand, 1979, p. 178)

Writing is not so much about saying something as it is about finding a process that will bring about new ideas, ideas that would not have thought of if the writer had not started to say them (Connolly, 1986).

The goal of writing in a content area is to help students learn the subject matter of that discipline (Miller & England, 1989). "Instead of using writing to test other subjects, we can elevate it to where it will teach other subjects, for in making sense, the writing is making knowledge" (Moffett, 1982, p. 235). 'Writing to learn' in a discipline means developing students' understanding of the concepts inherent in the discipline by developing their capacity to use the language of the field fluently (Connolly, 1986). The quality of writing itself may improve

incidentally, just as one's ability to carry a tune may be improved, incidentally, through joining in campfire songs.

Writing to think. The third perspective for viewing knowledge and education stems from Paulo Freire's concept of 'liberating education'. The ability to think critically separates people who are capable of making independent choices from those who are passive receivers of information (Fulwiler, 1982).

> Literacy that provokes critical awareness and desocialization will mean more than basic competency. It will be critical literacy across the curriculum, asking all courses to develop reading, writing, thinking, speaking and listening habits to provoke conceptual inquiry into self and society and into the very discipline under study. This means that future teachers in every subject , from biology to architecture, can study how their special competence can generally develop thinking and language skills. ... Critical literacy invites teachers to problematize all subjects of study, that is, to understand existing knowledge as a historical product deeply invested with the values of those who developed such knowledge. (Shor, 1987, p. 23)

Knowledge and reality are not fixed. They are socially constructed within a community (Rorty, 1969). The school's primary task is to change passive participants into active observers who understand what knowledge is, how it is made, and who can take part in its production (Connolly, 1986).

The approach assumes an intricate interaction among teachers, learners and social change (Elsasser & John-Steiner, 1987). When students are asked to communicate their

ideas, they are being told that their ideas are valued and that they are capable of expressing them. In communicating, students exercise greater power and control over their own learning.

> We have seen that using language can empower people, enable them to survive in body and flourish in spirit. We have seen how the force to limit communication - whether that force takes the form of monopoly in mass media or the radical narrowing of standards of "acceptable language" - can intimidate, passify (not pacify), and disenfranchise people. Yes, writing across the curriculum advocates want people to write about whatever they study, because they see writing as power, whether that power be political or spiritual or therapeutic or intellectual. (Thaiss, 1988, p. 92)

Writing is the specific act which most promotes independent thought (Fulwiler, 1986). Both the decision to write and the process of writing are actions; "one cannot be passive and at the same time generate words, sentences and paragraph - thoughts" (p. 25). The written text is available for review of ideas and thoughts. Writing forces one to be explicit, to organize and examine one's previously held ideas, and to develop new ones (Langer & Applebee, 1987).

> By writing what I think I know, I develop what I potentially knew. Writing does more than reflect underlying thought, it liberates and develops it. ... it is a superb instrument not only for exploring the potential of thought but for developing it as well. (Smith, 1982, pp. 33-34)

From this perspective, writing in content areas is not done to improve writing, nor to improve student's understanding of concepts in a discipline, although both of these outcomes may occur. The aim of writing in any subject is to enable students to think critically and independently.

Research investigating writing in content areas. In this section, studies examining writing activities in content areas will be discussed. As programs of writing in content areas tend to be grounded in one of the three paradigms described above, so do research studies in the area. Researchers situated in different paradigms tend to ask different questions.

Many studies, most at the post-secondary level, have examined the effects of writings on students. The types of questions addressed in student-oriented studies situated in each of the three paradigms will be reviewed. Research relating to teachers of writing programs will be described in more detail.

Several studies have tried to determine the types of writing activities most commonly used by teachers. Most studies report that the majority of student writing is transactional (Applebee, 1981; Bernhardt, 1985; Behrens, 1978; Donlan, 1974; McCarthy, 1987; Tighe & Koziol, 1982; Williamson, 1983). Most students do not get much exposure to writing in content areas (Selfe & McCulley, 1986).

A few studies did report teachers using writing expressively or for learning (Gorman, Gorman & Young, 1986; Zemelman, 1977). Briggs (1981) found that 62% of high school teachers do not link writing and learning. Most think all writing should be evaluated. Even teachers who said that writing should be used to assist students in learning did not carry this view into their classroom practice (Maimon & Nodine, 1978). Tighe and Koziol (1982) found that teachers' attitudes to writing could affect students responses.

Several studies found that participation in workshops gave teachers greater sensitivity to students' difficulties in writing and moved their perspective closer to a 'writing to learn' view (Blake, 1976; Fulwiler, Gorman & Gorman, 1986). The views of English teachers changed most. Those of mathematics and science teachers changed the least (Kalmback & Gorman, 1986).

Student-oriented studies tended to focus on the outcomes of writing programs. The first studies in the area of writing in content areas viewed writing as 'learning to write'. Questions of how best to organize writing programs in content areas to produce better writing and whether such programs improved students' writing or attitudes to writing predominated. Grammar and spelling were major concerns for teachers.

With the 'writing to learn' perspective, research started to focus on whether students did indeed learn subject area content better. Studies looking at retention and achievement were common.

Research based in the 'writing to think' paradigm examined the relationship between writing and thinking. Common concerns were whether writing could improve students' thinking skills, whether writing could empower students as learners and whether writing could affect students' attitudes to learning.

## Writing in Mathematics

In this section, types of writing activities described in the literature will be described. Some of the questions involved in implementing programs of writing activities will be discussed. Finally, research into the area of writing in mathematics will be reviewed.

Writing activities: Writing can be used in many different ways in the mathematics classroom. Some authors (Birken, 1989; King, 1982; Rose, 1989) categorize writing activities as either expressive or transactional.

Types of transactional writing activities, intended to inform, explain, report, or persuade, include:

1. summaries, which help students to synthesize content and make it more meaningful (Johnson, 1983; Johnson, 1990; ; Keith, 1986; King, 1982);
2. completion questions, which ask students to complete a partial sentence (Azzolino, 1990; Keith, 1988; King, 1982; LeGere, 1991);
3. 'lead' sentences, which provide a statement and require students to respond by completing a second statement on the same topic (Azzolino, 1990);
4. explanations, which encourage students to use more precise mathematical language (Geeslin, 1977; King, 1982; McMillen, 1986; Schmidt, 1985);
5. definitions, which force students to think about the meanings of terms (Keith, 1986; Rose, 1986);
6. debriefing, which asks students to list steps in a procedure or problem (Azzolino, 1990);
7. word problems, which encourage students to use clear, concise language in making up their own problems or in rewording problems from exercises (Johnson, 1983; Keith, 1988; Kennedy, 1985; King, 1982; Stempien & Borasi, 1985);
8. stories, which allow students to write creatively about mathematics (Rose, 1986);

9. projects, which allow students individually or in groups to research and present reports on topics related to mathematics (Abel, 1987; Keith, 1986; Sommers, 1992);
10. term papers, reports or essays, which get students to investigate non-standard topics in mathematics (Burton, 1985; Frankenstein, 1987; Geeslin, 1977; Johnson, 1983; King, 1982; Mett, 1987; Schmidt, 1985; Stempien & Borasi, 1985);
11. books, which encourage student to compile information on a mathematics topic for use in class (Montague, 1973);
12. note-taking, which gets students to record information from class presentations or text books (McMillen, 1986);
13. dialogues, which allow students to debate problem-solving activities or philosophical issues in mathematics (Stempien & Borasi, 1985); and
14. word banks, which provide students with a source of terms for use in writing (Azzolino, 1990).

Types of expressive writing activities, intended for the student to put his or her thoughts on paper, include:

1. journals, which encourage students to record their thoughts about mathematical concepts (Edwards, 1991; Frankenstein, 1987; Kennedy,1985; McMillan, 1986; Mett, 1987; Nahrgang & Petersen, 1986; Selfe, Nahrgang & Petersen, 1986; Shaw,1987; Vukovich, 1985; Watson,1980);

2. freewriting, or daydreaming, which allow students to think aloud on paper, quickly jotting down whatever comes to mind; (Burton, 1983; Burton, 1985; King, 1982; LeGere, 1991; McMillen, 1986; Mett, 1987);
3. letters, which require students to write to a close friend or relative about mathematical content (Countryman, 1987; Keith, 1986; Kennedy, 1985; King, 1982; Schmidt, 1985);
4. admit slips, written upon entry to the class, which encourage students to write their thoughts on paper for sharing anonymously later (Schmidt, 1985); and
5. autobiographical writing, which allow students to write about their own history and feelings (Rose, 1989).

There are three difficulties with the use of these categories in classifying writing activities (Powell & Lopez, 1989). First, they refer to the writer's intentions rather than the teacher's intentions. Second, many writing activities develop naturally from an expressive mode to a transactional one. For example, journals and letters can embody expressive writing, transactional writing, or a combination of both. Third, poetic writing is a largely ignored, but important type of writing. It is in poetic writing that the imagination is used.

> We wouldn't claim to understand fully what happens when children's imagination is brought into play, but in its widest sense we would regard imagination as that mental process which enables a person to make his own connections, whether this happens to be in the sciences or in the arts. It may be that those moments are rare when an `imaginative leap' opens up new patterns and new perspectives for others, but unless we provide many opportunities all over the curriculum for children to use their imagination more extensively, their knowledge will remain inert. (Martin, D'Arcy, Newton & Parker, 1976, p. 86)

For the teacher intending to use writing activities in the classroom, focusing on the subject of the writing activity is more useful. Thus, the full range of activities from expressive to transactional or poetic is possible.

Miller and England (1989) identified four categories of writing activities:

1. contextual activities, which are affective and student oriented;
2. instructional activities, which are focused on teacher-student communication about instruction;
3. reflective activities, which are used to promote analysis or clarification of concepts; and
4. miscellaneous activities, which are intended to involve the student with mathematics personally and creatively.

Programs of writing activities. Writing has been used at different points in the lesson (Azzolino, 1990). Teachers may ask students to write at the beginning of a class to review previously-studied material and make the transition to

a new topic (Miller, 1992). Others use writing during the class to obtain immediate feedback from students. Some use writing at the end of a class to review and summarize important concepts and procedures.

Different techniques for collecting student writings have been developed. Students can write responses on cards or on individual sheets that are quickly collected and reviewed during or between classes (Le Gere, 1991). Alternately, students can put all their responses in a journal or log book. When reviewed periodically by the teacher, changes in patterns of responses will be clearer (Borasi & Rose, 1989; Burton, 1983; Craig, 1983; Fulwiler, 1980; Nahrgang & Petersen, 1989).

Typical assignments are brief, focused, and take little time to do or to read. Simple, short prompts tend to result in more and better writing by students (Miller & England, 1989). In most studies, writing has been done in class (Azzolino, 1990; Borasi & Rose, 1989; Johnson, 1983; Le Gere, 1991; Miller & England, 1989), although occasionally writing has been included in homework assignments (Le Gere, 1991) and in tests (Azzolino, 1990; Le Gere, 1991; Johnson, 1983). Students are encouraged to respond in prose. The more mathematical symbols and notation are used, the less understanding is shown in responses (Nahrgang & Petersen, 1986).

Research has indicated that guiding students through the writing process is instrumental in producing better thinking and writing (Ford, 1990). Students need to be prompted carefully to begin writing. The use of active words like analyze, compare, contrast, explain, hypothesize, justify, relate, restate, summarize, and support are effective in giving specific direction to students. Keith (1988) suggests that students may need to see examples of written solutions in order to improve their own. Peer evaluations can be effective.

A variety of approaches have been used to evaluate students writing. Schemes ranging from reading everything thoroughly, to spot checking, to not reading anything at all have been tried (Azzolino, 1990). However, most researchers feel that teachers should read students' responses. This not only gives the teacher valuable information, but validates writing activities for students.

Several general trends in evaluation are apparent. First, the teacher should correct any mathematical errors in students' writing , but not grammatical or mechanical errors (Nahrgang & Petersen, 1986). Student writing should be approached as part of a process leading to completion in the future, rather than as a finished product (Keith, 1988). Second, teachers should make supportive, non-evaluative comments on students' writing (Borasi & Rose, 1989; Le Gere,

1991; Nahrgang & Petersen, 1989). Such comments communicate teachers' interest back to students and are important for motivation. Third, student writing should count for something (Fulwiler, 1980). Nahrgang and Petersen (1986) suggest adding one or two points onto the next major examination. Holistic marking schemes for essays have been designed by Keith (1988) and Burton (1983).

Not all researchers agree that writing should be marked. Azzolino (1990) points out the importance of remembering that writing is not an end in itself, but a means to learning mathematics and mathematical thinking. Martin, D'Arcy, Newton, and Parker (1976) note that changes occur in the process of writing when it is evaluated. Craig (1983), referring to journal writing in general, feels that it is inappropriate to mark the record of someone's personal development.

Research investigating writing in mathematics. While most of the literature on writing in mathematics is anecdotal in nature, a number of studies have been carried out. Pearce and Davison (1988) surveyed and observed junior high mathematics classes to determine the types of writing used in the mathematics. They found that the majority of writing was mechanical and passive in nature.

Weissglass, Mumme and Cronin (1990) examined how teachers could foster or hinder writing and verbal

communication in grades two and three mathematics classes. They found that the degree of control related to the amount and quality of communication between student and teacher. They also found evidence of teachers constructing their own understanding of teaching and learning.

White and Dunn (1986) tried to determine whether participation in a writing workshop could change high school mathematics teachers beliefs of mathematics and learning. They found that teachers' views had been expanded by the workshop.

Most research examining writing in mathematics focused on the effects of writing on students. Many of these studies were situated in the 'writing to learn' paradigm. Several investigated whether writing improved student achievement. Results were contradictory. In some, no significant differences were found between treatment and control groups (Bell, 1987; Burton, 1986; Miller & England, 1989; Paik, 1983; Selfe, Peterson & Nahrgang, 1986; Wells, 1986). Others found that treatment groups did achieve significantly better than control groups (Evans, 1984; Ganguli, 1989; Gladstone, 1987; Lesnak, 1986; Morrow & Schifter, 1988; Powell & Lopez, 1986). Johnson (1990) reported similar findings, but only when journals were checked regularly. Students with the lowest pre-test scores tended to gain the most (Evans, 1984; Wells, 1986).

Bell and Bell (1985) and Bell (1987) found that expository writing increased students' problem solving skills. Wolff (1985) and Burton (1986) found that writing improved students' retention of mathematical knowledge, particularly for high-ability students.

Other research was grounded in the 'writing to think' paradigm. Burton (1986) reported that students who had experience writing in mathematics became more confident and had a better attitude to algebra. Other researchers confirmed this finding (Buerck, 1986; Lesnak, 1986; Miller & England, 1989; Morrow & Schifter, 1988; Powell & Lopez, 1986).

Wason-Ellam (1987) reported that expressive writing makes learning active and personal. Edwards (1991) found that students began to make connections between new material and prior learning, and began to take more control of their learning.

Other studies explored whether writing helped students to develop thinking skills. Linn (1987) found that writing can enhance students metacognitive abilities. Williamson (1991) used journal writing with cooperative learning to develop metacognitive skills in eleven-year-old mathematics students. She found that average and below-average students received more benefit from the activities.

These studies and many anecdotal articles report a number of benefits accruing to the student from the use of writing activities in mathematics classrooms. Writing in mathematics benefits students in a number of ways.

1. It has a therapeutic effect, allowing students to express their fears, frustrations, and attitudes about mathematics and schooling in general (Borasi & Rose, 1989; Buerck, 1986; Lesnak, 1986; Morrow & Schifter, 1988; Sommers, 1992).
2. Writing can lead to better understanding and improved mastery of mathematical content (Buerck, 1986; Burton, 1986; Evans, 1984; Ganguli; 1989; Gladstone, 1987; Haley-James, 1982; Lesnak, 1986; Morrow & Schifter, 1988; Robertson & Miller, 1988; Sommers, 1992; Wolff, 1985).
3. Many of the skills needed for problem solving are practiced when writing (Azzolino, 1990; Bell, 1987; Geeslin, 1977; Johnson, 1983; LeGere, 1991; Linn, 1987; Keith, 1988; Williamson, 1991).
4. Writing offers students the opportunity to bridge the gap between real world phenomena and abstract thinking (Borasi & Rose, 1989).
5. Through writing students become more aware of how they do mathematics and develop metacognitive skills (Azzolino, 1990; Borasi & Rose, 1989; Johnson, 1983;

LeGere, 1991; Linn, 1987; Miller & England, 1989; Pimm, 1987).

6. As students write they discover that they are able to enquire, discover, and reflect in order to take control of their own learning (Edwards & Walker, 1990).
7. Writing also may enable students to move from the relatively informal speech of natural language to the very formal written language that is seen to characterizes mathematics (Brown, 1991; Johnson, 1990; Pimm, 1987).
8. Writing may be of assistance in broadening students' conceptions of mathematics (Borasi & Rose, 1989; Johnson, 1990; Schmidt, 1985; Sommers, 1992).
9. Writing activities give students the opportunity to experiment with and create mathematics on their own (Azzolino, 1990).
10. Writing can help to put the 'play' back into mathematics. Fellows (1991) notes that the word 'fun' is missing from any discussion of mathematics education.

> This is a significant omission in a world that overwhelmingly relates to mathematics with fear and loathing, while the people who do mathematics research are having so much fun with such wild and crazy stuff. (p. 12)

Fewer studies have been done to consider the effects of writing activities on teachers or on the classroom

environment as a whole. Miller and England (1989) found that writing activities may be of even more use to teachers than to students.

Borasi and Rose (1989) and Rose (1989) carried out two major studies examining the benefits deriving from programs to writing activities for teachers. Their suggestions with those of other writers and researchers are reported below.

1. Writing acts as a record of students' thinking (Fulwiler, 1980), and thus provides an excellent diagnostic tool for teachers (Borasi & Rose, 1989; Edwards, 1991; Nahrgang & Petersen, 1986; Rose, 1989).
2. Information gained from students' writings may lead to immediate changes and improvements in the course as the teachers sees the need to review, enrich, or provide remediation to students (Azzolino, 1990; Borasi & Rose, 1989; LeGere, 1991; Miller & England, 1989; Rose, 1989).
3. Writing activities have the potential to give teachers important knowledge of their students' feelings and attitudes (Borasi & Rose, 1989; Rose, 1990).
4. Teachers may become more aware of students' use of mathematical language, thus enabling the teacher to guide the student toward more conventional use of mathematical expressions (Brown, 1991).
5. Writing activities may provide sensitive means of evaluation (Pimm, 1987).

6. Teachers may become more aware of their conceptions and feelings toward teaching and mathematics (LeGere, 1991). This may lead to long-term changes in instructional approaches (Borasi & Rose, 1989; Rose, 1989).
7. Writing can be collected as a record of work to show to parents or as a record of the teacher's success in teaching (Pimm, 1987).
8. Writing is a silent activity, more easily controlled that many other classroom activities (Pimm, 1987).
9. While students are writing, the teachers has time to take attendance or do other chores (Miller, 1992; Pimm, 1987).

Researchers have also noted positive changes in classroom environment due to the use of writing activities. Suggested benefits are reported below.

1. There may be an increase in the individualization of instruction as teachers are able to diagnose student difficulties more quickly and more effectively (Borasi & Rose, 1989).
2. There may be an increase in the level of trust and mutual respect between student and teacher (Keith, 1988; Rose, 1989; Schmidt, 1985).
3. The classroom may become a less threatening place for students as the focus shifts from competitive mastery to

cooperative processing and interpretation (Borasi & Rose, 1989; LeGere, 1991; Miller, 1992; Rose, 1989).

4. A lessening of anxiety may result in a motivating effect on both teacher and student (Borasi & Rose, 1989).
5. Writing demands greater involvement from students than does small or large group discussion (Le Gere, 1991; Azzolino, 1990).
6. Responsibility for learning is shared by students and teacher (Le Gere, 1991).
7. Changes may occur in students' and teacher's attitudes to each other (Miller & England, 1989).
8. Writing provides the teacher one-to-one communication with each student, something often not easily accomplished through regular classroom interaction (Pimm, 1987; Rose, 1989; Schmidt, 1985).

Summary

Writing has a number of unique qualities, not shared by other forms of communication. Writing is naturally reflective and involves the hand, eye, and brain. These attributes provide for a special relationship between writing and learning.

Writing in content areas can be viewed from three perspectives: 'learning to write'; 'writing to learn'; and 'writing to think'. These perspectives are similar to

Habermas' three paradigmatic orientations as outlined by Aoki (1979). 'Learning to write' is clearly situated in the empirical-analytic orientation. Emphasis is on control and predictability. Schools act upon students. 'Writing to learn' has a situational-interpretive orientation. Communication and personal construction of knowledge are key activities. "Understanding is in terms of meanings people give to situations." (Aoki, 1979, p. 8) Both the student and his environment are related and affected by each other. 'Writing to think' involves reflection from a critical-theoretic orientation. The focus is on "liberating" students to be independent learners, to examine hidden assumptions, and to think critically. The aim is to empower students, enabling them to transform their world (Aoki, 1979).

The majority of the literature concerning writing in mathematics deals with the effects of writing programs on students. Many affective and cognitive benefits to students have been identified. Such findings indicate that writing meets the minimum criteria for inclusion in the mathematics classroom.

Since research has established that writing benefits students, it is appropriate to examine potential benefits to teachers. Little research has been done in this area. In particular, the effects on the teacher of reading students' writing have not been investigated carefully. Many

suggestions have been tentatively put forth by researchers, but a systematic, in-depth study of teachers' perceptions of the value of writing activities to them has not been done. This study will explore whether writing activities in mathematics can enhance teachers' awareness and reflection in the classroom.

# Chapter III - Design of the Study

## Introduction

The purpose of this study is to explore whether a program of writing activities can provide teachers with the opportunity to become more aware of:

1. the effectiveness of classroom instruction;
2. students' attitudes and feelings;
3. students' understanding of mathematical content; and
4. teachers' own conceptions of mathematics and teaching.

In order to address these questions, four teachers were invited to use writing activities in their Mathematics 30 classes. Prompts were developed by the researcher to match the objectives of units being studied. Participants experimented using writing activities during the first phase of the study, in November and December of 1991. Based on teachers' experiences during this developmental phase, revisions were made in plans for the second phase of the study. During this implementation phase, which took place in February and March of 1992, teachers developed their own programs of writing activities as part of their Mathematics 30 courses.

Setting of the Study

The study was carried out at a large, urban high school with over 2000 students. The school draws from areas of middle and high socio-economic status. Students are of mixed ethnic and racial background.

Mathematics 30 is the final course in the academic stream of high school mathematics in Alberta. A new curriculum for the course, mandating the use of writing activities, was introduced for the 1991/92 school year (Alberta Education, 1990a, 1991b). Writing activities were to be part of instruction and part of the final Diploma examination starting in January, 1992 (Alberta Education, 1990b).

These changes in curriculum provided a unique opportunity to become aware of changes in teachers' perceptions about both the course and the students. In a course taught many times before, teachers come to a gradual awareness of students' attitudes to the material in the course, effective instructional strategies, and problems typically experienced by students and teachers. In a new course, all of these understandings need to be re-evaluated. Thus, teachers might be more open to new perceptions.

The four teachers who participated in the study included two experienced and two relatively inexperienced teachers of Mathematics 30. The 'novice' teachers, Ian and Katherine,

were selected because of their inexperience and the consequent possibility that they might be particularly aware of information gained from writing activities. The 'expert' teachers, Gordon and Mary, were in a position to perceive whether different feedback was made available through student writing.

Ian met with his full-year Mathematics 30 classes three times a week for sixty-seven minutes a period. The other three participants taught semestered classes, meeting with each group of students six times a week for sixty-seven minutes per class.

Writing Prompts

Shortly before each phase of the study, teachers indicated the units that would be covered. The researcher prepared several writing prompts for each objective in these units, according to the new curriculum (Alberta Education 1991b). A number of prompts not specific to any particular objective were developed for general use. These related to student attitudes, metacognitive skills, or classroom instruction in general. These writing prompts were classified according to the type of feedback they might provide for teachers. The categorization is based on that designed by Miller and England (1989).

1. Content writing activities were used to assess students' understanding of mathematical concepts and processes. They were also used to promote analysis and clarification of concepts and skills, including metacognitive strategies.
2. Affective writing activities were students oriented, and had the potential to give teachers important knowledge of their students' feelings and attitudes.
3. Instructional writing activities were focused on teacher-student communication about instruction. Such activities had the potential to give teachers valuable feedback on their teaching and on the effectiveness of different instructional strategies.
4. Creative writing activities were intended to involve the student in mathematics personally and imaginatively. Such activities might catch students' attention, provide enjoyment, and develop broader understandings of mathematics.

Writing prompts for the units to be studied were provided to participants. Teachers were free to select those prompts they deemed appropriate for use in their classes. They were also encouraged to develop and share their own writing prompts. Writing activities developed by participants were added to the collection of prompts.

Teacher Inservice

Before the study began, all four teachers attended an inservice on uses of writing activities in mathematics. Ways in which writing activities could be employed effectively were described. Samples of different types of writing prompts were presented. Possible student reactions to writing and potential problems were discussed. Possible benefits to students and teachers were reviewed.

During the inservice, teachers inquired whether writing activities should be graded. The relative merits of doing so were discussed. The making of brief notes on student responses was suggested as an alternative.

Each participant received a short handout containing the information mentioned above. All participants were given the opportunity to ask questions about the use of writing activities and the study itself.

Developmental Phase

The developmental phase took place over four weeks in November and December, 1991. Teachers were asked to have students write for five minutes in at least half of their class periods.

Writing prompts were provided, ready for duplication, one per page. Teachers duplicated those chosen for use, distributed them one at a time to students, and collected

them after writing was complete. Responses were kept by teachers until they could be reviewed by teachers and the researcher.

Once a week, informal, audio-taped interviews were conducted during which teachers were asked to comment on their impressions of student responses and on the information contained in them. Input was solicited on ways in which writing prompts could be made more effective and on how best to incorporate writing activities in their mathematics classes. Copies were made of all student responses, before they were returned to students.

Teachers were encouraged to keep journals themselves, writing in these when students wrote. Ian started to keep a journal, but soon stopped. He found that he did not have the time needed to continue. The other participants did not begin to write journals, again citing lack of time.

At the end of the developmental phase, teachers were asked to summarize, orally and in a brief questionnaire, their opinions about writing in mathematics and the information it gave them.

The developmental phase served a variety of purposes:

1. It provided an opportunity for refining writing prompts developed by both researcher and participating teachers.
2. It allowed teachers to experiment with the conditions under which they could incorporate writing activities in

the classroom. Factors of frequency, time, purposes, placement in class, format, and evaluation were considered.

3. It allowed participants to become accustomed to the types of student-teacher interactions that occurred through writing activities.
4. It allowed the researcher to note the types of awareness and understanding participating teachers gained from reading students' writings.

Implementation Phase

Three participants, Katherine, Ian, and Mary, were involved in the second phase of the study. The fourth teacher, Gordon, was assigned a student teacher during this period and was, thus, not able to continue. Different classes of Mathematics 30 students were used to increase the chances of teachers becoming aware of differences in feedback during the two phases.

Ian decided to continue using writing activities with his phase one class. Thus, he had two classes involved in the implementation phase.

Prior to the commencement of the implementation phase, the researcher and participants discussed possible changes for phase two. Although different strategies were discussed, teachers decided to continue putting writing prompts on

separate sheets, ready for duplication. Teachers found that they could not do writing as often as initially hoped. One writing activity every three classes was proposed as being more feasible.

Writing prompts were again prepared by the researcher. Prompts were duplicated by the teacher, distributed for students to write on, and then collected. The researcher held weekly, informal, audio-taped interviews to discuss the information teachers gained from student writing and their reasons and strategies for using writing activities.

At the end of the study, teachers were asked to summarize, orally and in a brief questionnaire, their perceptions of the value of writing in mathematics and their opinions as to the best use of writing activities in mathematics.

At this time, students were also asked to write about their impressions and opinions of writing in mathematics. Based on their views, seven students, selected by the researcher after discussion with the teacher, were informally interviewed. Some saw writing as a valuable part of mathematics: others did not. They represented both male and female students, with marks in mathematics ranging from 40% to 90%. Several English-as-a-second-language students were included in this group.

The implementation phase served a variety of purposes:

1. It allowed teachers an opportunity to integrate writing activities into their programs of instruction from the beginning of the course, or, in Ian's case, immediately after mid-term examinations.
2. It allowed teachers to develop their own programs of writing activities.
3. It allowed the researcher to observe and collect data on the ways in which participants set up and viewed programs of writing activities.
4. It allowed the researcher to continue to observe and collect data on teachers' awareness of information about and from students.
5. It allowed researcher and participants to become more aware of students' views of writing in mathematics.

The emphasis of the study changed in phase two. In the developmental phase, interviews focused primarily on teachers' perceptions of students' understandings of mathematical content, students' attitudes and feelings, and the effectiveness of classroom instruction. The second phase of the study, therefore, focused on the question of whether writing activities provided opportunities for teachers to reflect on their conceptions of mathematics and teaching.

## Data Analysis

Data from teacher interviews, their oral summaries, and questionnaires were categorized according to the type of information they conveyed. Keeping in mind the four questions upon which the study was based, four categories of teacher comments were defined:

1. understanding comments, related to teachers' perceptions of students' understanding of mathematical concepts, skills and processes;
2. attitude comments, related to teachers' perceptions of students' attitudes and feelings to mathematics, the Mathematics 30 course, mathematics teachers, or various topics studied;
3. teaching comments, related to teachers' perceptions of students' feedback on the effectiveness of particular teaching techniques or classroom instruction in general; and
4. reflective comments, related to teachers' reflections about mathematics, writing in mathematics, teaching, and learning.

## Summary

Through both phases of the study, teachers experimented by using writing activities and then integrating writing activities into their Mathematics 30 courses.

Data were collected from a variety of sources, primarily weekly interviews with participants. Teacher comments from these data were analyzed and related to the four questions upon which the study was based. In the following two chapters, the researcher will attempt to answer these questions. Do programs of writing activities assist teachers of mathematics to:

1. become more aware of the effectiveness of classroom instruction;
2. become more aware of students' attitudes and feelings;
3. become more aware of students' understanding of mathematical content; and
4. reflect on their own conceptions of mathematics and teaching?

In chapter IV , the first three questions will be addressed. Feedback in these areas can be perceived directly by teachers or inferred from student responses. The last question involves more than teachers' awareness of students and classroom. Reflection requires self-awareness, as well. Therefore, the fourth question will be examined, separately, in Chapter IV.

# Chapter IV - Listening to the Student's Voice

## Introduction

Some researchers have mourned the fact that the teacher's voice is often unheard in education (Provenzo, McCloskey, Kottkamp, & Cohn, 1989). The student's voice is even less audible in schools. Just as teachers need to be heard, "children also need to be listened to, without critical or negative judgment" (Van Manen, 1991, p. 86). and writing activities can provide a way to give students a chance to speak.

This chapter examines some of the areas in which teachers may, thus, learn from their pupils. Findings are presented to show that student writing may assist teachers to become more aware of:

1. the effectiveness of classroom instruction;
2. students' attitudes and feelings; and
3. students' understandings of mathematical content.

In investigating these three themes, data from both phases of the study were examined in a cross-case analysis. Teachers discussed writing activities used with Mathematics 30 classes, and occasionally with other groups of students, in weekly interviews and written questionnaires given at the end of each phase of the study. Comments revealing their perceptions provided the majority of the information

analyzed. Data from students' written responses, student interviews conducted at the end of the study, and observations made by the researcher were also examined.

In exploring the data, it was necessary to determine whether:

1. student responses contained the types of information that could serve as a source of feedback for teachers;
2. teachers demonstrated their awareness of information contained in student writing by making explicit statements in teacher interviews or questionnaires; and
3. teachers showed implicitly that they had internalized this awareness by making changes in their attitudes or teaching as they responded to feedback.

Data related to each of these points will be discussed in the sections below.

## Teachers' Awareness of the Effectiveness of Classroom Instruction

Comments revealing teachers' awareness of the effectiveness of classroom instruction fell into five categories. These related to feedback in student writing on the effectiveness of:

1. general mathematics instruction;
2. different teaching techniques;
3. instruction on specific topics;

4. classroom communication; and

5. instruction on writing.

Student writing provided information to teachers in each of these areas, as described below.

General mathematics instruction. Students had strong opinions about good mathematics teaching. One prompt was particularly useful in getting this type of information.

> Prompt: Make believe that you are writing to a previous math teacher. What would you tell them you wish they had done more of? What would you tell them you wish they had done less of?

Other information came from student responses to a variety of other prompts.

Students had many opinions about what they saw as good mathematics instruction. They liked to see:

1. more review questions;
2. more classroom discussion and interaction among students;
3. more definitions given in class;
4. more homework checks;
5. more practice questions;
6. more explanations;
7. better notes given in class;
8. more class time allowed for work;
9. more relevant, real-life examples; and
10. less yelling and sarcasm.

While a wide variety of good, thoughtful suggestions were made, a common thread runs through them. Students were asking teachers for more guidance, for more support in their learning. They saw this support as an important factor in their learning. Their suggestions were ways of making instruction more effective, and formed a scarcely-veiled message to teachers.

Teachers reacted to this information in a variety of ways. Some feedback was well-received. One teacher found the following student response in a debriefing question after a test. He acknowledged that the student's view was probably justified.

> Student: Mr. _____ , you're a good teacher, but you should smile sometimes. Sometimes you kind of intimidate people when you don't smile. Sometimes, they're afraid of you. (4, AC, Nov. 25, 1)
>
> Teacher: I probably should relax and smile more. I may appear grim. (4, Nov. 25, 1)

Another teacher cheerfully reported that she and her class had exchanged views on what they saw as problems in the course. Her students wrote that they felt she was too impatient with them. She appeared to take what they said seriously.

The feedback in the responses, above, was personal. Other responses, however, were of a more general nature.

> Student: In looking back at my previous math course, I wish you had been more direct in your explanations. Quite often, our examples in class were very simple, yet exam questions were definitely harder. Perhaps if more types of questions had been shown to the class, the exams would not have seemed so strange or new. Asking questions was not encouraged as the student often seemed to be humiliated for not understanding concepts. Less talk and more help was required. (1, AS, Nov 29, 2)

> Teacher: Kids are afraid to ask questions. I don't know if that's me or in the past. They must have had a teacher who humiliated them, told them they were stupid. (1, Nov. 29, 2)

While the student's response was general and ostensibly referred to a previous mathematics teacher, the teacher wondered if the message was for her. The student may have written in response to a specific incident involving the student himself or another student in the class. Some teachers appeared to feel that this was the case and such general comments often were not well received.

> Teacher: A question like this relieves the responsibility of the student - that's the impression I'm getting right now. The teacher should have done this or that. (3, Nov. 27, 1/2).

Teachers seemed to feel that they were being blamed for students' success or lack of success in mathematics. In addition, teachers had doubts about whether student suggestions could be accommodated practically.

> Teacher: If someone can tell you they like to work in pairs, or in groups, or they like more examples ... and you see a group consensus, then implementation is easy. For individuals, a short one-to-one interview can ... accommodate some aspects of requests or a compromise could be worked out. (2, questionnaire, phase 1)

> Teacher: What one (student) likes another may not. For example, one likes lots of examples, another wants fewer and more time to work on their own. (1, questionnaire, phase 1)

There was, however, some indication that teachers did reflect and act on the students' messages. One teacher, almost reluctantly, agreed with a student's suggestion to give more complete notes in class. Whether the change actually took place is not known.

> Student: I wish you could have spent more time teaching the definitions of mathematical terminology. I find math is not so difficult in the practical sense of working out a problem, ... yet I am very confused when the time comes (to) understand terms, words. (3, CR, Nov. 27, 2)

> Teacher: She doesn't write down definitions (in class), but her first statement is that she wants more definitions. ... She'd rather (complain) than (do) math. ... Maybe I have to start writing stuff on the board and get them to memorize. (It) seems like memorizing anyway. ... (This writing question) is good for me because I'm looking at what I'm actually doing, more so than I might (otherwise). (3, Nov. 27, 2)

It appears that, in some ways, teachers blame students for ineffective instruction, just as students blame teachers. Neither party seems to acknowledge instruction as a joint venture.

Whether much change in instruction took place as a result of student writing is not clear. Such changes are, perhaps, more likely to occur over the long term. But one change clearly occurred. Teachers listened to what students said about instruction in mathematics. Perhaps this is as important as any other change that might occur. The cry for

more support would seem to reflect a lack of confidence in students' ability to do mathematics. Paying attention to them or even acknowledging their concerns might do much to overcome their anxiety.

Teaching techniques. Student writing did provide a source of information about the effectiveness of different teaching techniques. Sometimes this information was stated explicitly in student responses. One teacher specifically used writing to get feedback on a particular lesson.

> Teacher: We had a 45 minute quiz today. After that we picked up the graphing calculators (and worked together in pairs). ... Since it was a different kind of class, I wanted to see what their reaction was.(2, March 2, 9)
>
> Prompt: What did you like from today's class? What did you not like from today's class?
>
> Student: The calculators were neat but very tedious. Its easier to learn because we were doing something rather than sitting and listening. ... One bad thing though, if we get used to the calculator we won't be able to do it on our own when we don't have one. (2,TS, March 2, 9)
>
> Student: I like using graphing calculators, but not in a group because my partner doesn't try, just copies. So I don't think it's fair. But I would appreciate if nothing was said. (2, BC, March 2, 9)
>
> Teacher: They all concentrated on the activity, but when it said, "did not like", they all concentrated on the test. (laughs) ... Certainly, there was something from the activity that (the students) didn't like either. I would have liked feedback on that. ... Some did (give such feedback.) ... One kid didn't like her partner. She says she sits there and does nothing. ... The kids generally enjoyed working with calculators. They could explore and see for themselves what the graph did. They enjoyed the chance to use the calculators. (It was a) change of pace. (2, March 2, 9)

Student responses gave the teacher something upon which to reflect. She thought carefully about using graphing calculators in another activity.

> Teacher: I think ... when it comes to actually having to graph something by yourself, it becomes a different matter than just looking at the calculator. ... "What did you learn? What's happening here?" (The students) like the pictures that come up on (the graphing calculators.) "But what parts do you look at to get some of the information out of it?" ... They're pretty pictures you move around but ... to really think ... I don't think some kids can do that on the graphing calculator. I think (working with one) generalizes the pattern. ... If you have one (graphing calculator), you can graph (a function), but ... they can't rely on it. (2, March 26, 14)

On other occasions, teachers were able to infer the effectiveness of teaching techniques by considering students' responses as a group. One teacher reconsidered his presentation of a topic in combinatorics.

> Prompt: What is the hardest thing about solving problems involving permutations and combinations?
>
> Teacher: I try to do contrasting examples. ... And I try to have almost the same problem. But I think a person would have to do a large number of examples like that before it would start to sink in. Just doing one contrasting situation like that is almost not worthwhile. Some of them will pick it up, but for the majority, it still sails over their heads. And yet its hard to know where to draw the line. Is it better for these kids to spend their time working on homework or is it better for me to give them lots and lots of examples? ... I don't know. ... I don't know the answer to that one. ... Maybe I'll have to spend some more time distinguishing between permutations and combinations because obviously it's a source of problems, trouble. I need to find better examples or do some more examples to help them understand what the difference between the two is and how to distinguish (between them).(4, Dec. 19, 5)

By comparing writing to other instructional activities, teachers were able to evaluate the effectiveness of these techniques, including that of writing itself as a teaching tool.

> Teacher: I'm getting to feel that the ability to write this out is even better than some of the applications. ... If you can express yourself and think of all the parts that lead in together I think you really know what you're doing. With applications - not really - by the time you wade through all the different things you can lose sense. (2, Dec. 9, 5)

> Teacher: I'm in a quandary (about) how to get the students to review, instead of the teacher doing all the work. My attempt (with this writing activity) was for them to put it together, instead of me. ... I feel that it was valuable for pulling ideas together, rather than (doing) more drill. This might be a better exercise (than drills and review exercises) at lots of points throughout (the course). (2, Dec. 9, 6)

Teachers did more than react to feedback on their teaching techniques. Upon reflection, they incorporated ideas emanating from students into their repertoire.

> Teacher: Well, when I read through this (student's response), I thought, ... "What is he doing here? ... I need to know this." ... I thought it was something I should take up with the class because it worked well. ...I said to the class, "I've never seen this (before)." ... But I think that it was really neat and that it obviously worked well and if they wanted to use it, fine. ... On the last quiz ... there were 3 or 4 kids who used it. (1, Nov. 29, 1)

This teacher used the technique again in the second semester.

Another participant was introduced to a useful analogy by one of his students. He incorporated this in his presentation of arithmetic sequences.

> We developed this amazing formula for an arithmetic sequence. ($t_n = a + (n-1)d$) "So why is it an n-1 and not an n? Why do we subtract 1?" I said, "Write me a couple of sentences why." ... Some phenomenal guesses. ... So then we talked about it and one girl ... came up with the analogy, "Is that sort of like, when you want three pieces of wood, you only make two cuts?" ... She said, "This is probably stupid." ... I said, "That's not stupid at all, that's excellent. I'm going to use that." So I used it. (3, Feb. 26, 7/8)

While feedback could come from a prompt specifically designed to elicit feedback on teaching techniques, most perceptions were generated by participants' own reflections on student responses. Teachers showed no reluctance to consider such changes in their use of teaching techniques.

A class's general weakness in an area enabled teachers to evaluate teaching techniques used in that area. Different methods of analyzing situations or solving problems caused teachers to re-evaluate their usual methods and to incorporate new ones. Teachers compared writing activities with other instructional techniques, deciding which might be more effective to use in which circumstances. Student writing proved to be a useful tool for assisting teachers to refine their use of teaching techniques. Teachers found it very useful in this respect.

Instruction on specific topics. Writing activities did help to assist teachers in becoming more aware of the effectiveness of instruction on specific topics. On occasion, it became apparent from student responses that instruction of subject material had not been effective. This

instruction of subject material had not been effective. This awareness came, not so much from individual student responses, as from the teacher's general impression of student understanding. The effectiveness of instruction was inferred by the teacher rather than stated by students.

Some teachers used writing prompts deliberately to check on students' understanding of a mathematical topic. They saw this understanding as reflecting on the effectiveness of their presentation of the topic.

> Teacher: I've done this a couple of times: "Before I go on, I want to make sure you guys understand this."... I did that with arithmetic sequences ($t_n = a + (n-1)d$). "Why the heck is it n-1?" Well, no one knew even though I'd just talked about it. So then, let's deal with it now and I got them to write. ... I had to go over it again. (3, April 22)

Teachers recognized the value of such feedback and the potential need for change in instruction.

> Teacher: (Are instructional prompts useful?) In that it forces them to say something they might not say face to face, yes. Useful to me in that I can try to adjust or revise my teaching methods ... (1, questionnaire, phase 1)

> Teacher: Yes, (questions about the effectiveness of instruction) are very useful to me so that I can adjust my instruction after seeing what worked or didn't work effectively. (1, questionnaire, phase 2)

> Teacher: (I'm) not sure about the effectiveness of specific techniques. If it were a complete/partial "bomb" then I would seriously look at what I did or said. (2, questionnaire, phase 2)

> Teacher: Knowing that a small portion of the class understands a topic may necessitate review before going on. (3, questionnaire, phase 2)

> Teacher: I think (writing) is (giving information on your teaching). ... If no one can answer a question after ( you talk) for 60 minutes, then obviously you weren't talking about the right things. (3, April 22)

Some participants did reteach topics, reacting to students' messages about the effectiveness of classroom instruction.

> Teacher: Most of them are telling me that a geometric (sequence) is like an exponential (function) and most of them are telling me that an arithmetic (sequence) will be like a straight line. ... I didn't talk about (the graphs being discrete rather than continuous). ... I'm not thinking it's an important thing. I wouldn't draw them a graph and show them things. (3, March 18, 9)

Apparently, the teacher reconsidered. The next day he gave his students a handout sheet illustrating the discrete graphs of different types of arithmetic and geometric sequences.

Teachers did think, however, that too much credence could be given to an individual student's response.

> Teacher: If you ask Bobby about this topic and he was daydreaming - now does it indicate that I was doing a lousy job teaching or that he wasn't paying attention. (3, April 22)

Participants could recognize when students did not understand topics and, thus, when instruction had not been effective, but pointed to external factors as reasons.

> Teacher: Guess what I found out! The kids find problems involving permutations and combinations really tough. ... I really think part of the problem is we're having to do it so quickly. I think it's partly lack of time. I think (that with) something like permutations and combinations, (you) need to play with (them) for a while before you feel comfortable. ... We're trying to teach the essence of permutations and combinations in a week and a half and that just is not really an adequate

> amount of time to do that. ... Maybe next time around, we'll try and provide a little more time. (4, Dec. 19, 5)

This theme was echoed by other participants. Teachers saw lack of time as preventing them from reviewing or reteaching topics.

> Teacher: Another time through, yes. (I would make changes in instruction.) This question was given after the topic was taught. We don't have time to go back and reteach it just because some kids have trouble. But it will have an effect the next time I teach it. ... No doubt it has an effect. It's just that it's not going to happen right now, because we're already past the topic. (4, Dec. 19, 5)

> Teacher: I found a lot out. I wish I'd done better. It is not easy to use as a guide to what I could change. We have to go so fast. (1, Dec. 18)

Although teachers indicated that changes in instruction might be made the next time the course was taught, no one had a systematic way of remembering or recording such changes. Several proposed changes did not occur, because teachers forgot that they intended to make them.

Writing activities did provide teachers with information on the effectiveness of instruction on specific topics. But this information came from teachers' perceptions of a class's understanding as a whole rather than from a student's individual comments. Participants hesitated to rely on an individual's response in judging whether instruction had been effective.

Some teachers did re-teach topics in response to student feedback. Others planned to make changes in instruction, but

did not. Even when class understanding as a whole was weak, participants thought the course went too fast to permit re-teaching of a topic. Teachers tended to attribute ineffective instruction to lack of time.

Classroom communication. Classroom communication has an effect on the effectiveness of classroom instruction. Writing activities contained information on the extent and adequacy of classroom communication. This gave teachers feedback that could have been useful in restructuring their instruction.

Teachers became aware of student-student interactions as they read students' written responses. On nine separate occasions, student writing told teachers that students were working together in class. This was not necessarily seen as a problem.

> Teacher: Read these two together. ...
>
> Prompt: What is the hardest thing about solving problems involving permutations and combinations?
>
> Student (Alex): It is hard to imagine all those possible combinations and to comprehend the mind-boggling numbers which exist. It is difficult to understand exactly what the question is asking. Its Just too Hard. (4, AC, Dec. 16, 5)
>
> Student (Terry): It is hard to imagine all those possible combinations. And to comprehend the mind boggling numbers which exist. It is difficult to understand what exactly the question is asking. ITS JUST TOO HARD! (4, TM, Dec. 16, 5)

> Teacher: Those guys think alike don't they. (laughs) I think (they wrote it together). I'm sure they did. My guess would be Terry copied from Alex. (4, Dec. 16, 5)

Participants also became more aware of student-teacher communication. One teacher was alerted to a potential problem through writing.

> Prompt: Are you happy with your mark so far? What can you do about it?
>
> Student: Also I make mistakes on quizzes (which are written on the board), because your 2 looks like a 3 and your 5 looks like a 3. (4, NY, Nov. 25, 1)
>
> Teacher: Like the student who confuses 2, 3, and 5, and complained about my writing. He sits at the back. He may need glasses or to be moved to the front (4, Nov. 25, 1)

Teachers recognized that writing enhanced the amount of communication in the classroom and found this of great value.

> Teacher: I want to know if students are not getting something they need. Students may be more likely to share their concerns in this way (through writing). (4, questionnaire, phase 1)

Teachers' perceptions of the effectiveness of classroom communication were enhanced by individual student responses. Information was limited, but potentially useful for reorganizing the class. The teacher might choose to separate students copying from each other or permit student to continue to work together if desired. Students with visual problems could be moved closer to the blackboard.

Instruction on writing. Student responses were of varying quality. Even teachers who viewed writing as expressive in nature, saw a need for helping students to

learn how to write, to organize, and to best present written responses. Throughout the study, participating teachers gave feedback to students in response to this perceived need. In a recursive manner, writing activities provided information about the effectiveness of instruction given in these areas, and made teachers reflect on ways to improve students' writing.

In both phases of the study, participants gave students guidance as to the format of written responses.

> Teacher: They need a little more structure on how to reply. ... I think they need that. ... At the beginning, when learning, ... let them write the first one and say, "Now, here are some guidelines, so you can learn each time you write." (2, Dec. 9, 5)

Students were seen to incorporate this feedback in their written responses. The comment, above, referred to a writing activity comparing geometric and arithmetic sequences. The teacher subsequently went back to her class and talked about how to present comparisons clearly. She used a particularly clear student response which displayed information in two parallel columns. On a similar question given later in the term, she commented that students had incorporated this suggestion.

> Prompt: Write out your own definition of a finite geometric series. Write out your own definition of an infinite arithmetic sequence. Imagine that you are trying to explain the difference between these to a friend.

Student:

| finite geometric series | infinite arithmetic sequence |
|---|---|
| 2+4+8+16+32+64 | 2,4,6,8,10,12,... |
| $t_2/t_1 = r$ | $t_2 - t_1 = d$ |
| Is a geometric series which is <u>not</u> infinite and the number of terms are known. | Is an arithmetic sequence in which the number of terms is not known and is infinite. |

<u>The difference is that</u>:
-one is infinite
-the number of terms is known for one and not known for the other

> Teacher: Remember the one we did where we said, "Use a parallel structure." He obviously did what we said. (2, DK, Dec. 17, 7)

All participants showed exemplary solutions to their students, using an overhead projector or distributing copies of a particularly complete or well-organized student response to the class. Teachers displayed a natural tactfulness by using students' ideas for this, rather than their own. Often good ideas were compiled from several student responses.

> Teacher: And so, then I put down how they might have presented it, ... "Well, split it up into two columns", as one or two people did", I said. I tried to always say that. "One or two of you actually thought of this. And another one or two of you, not the same people, thought of this idea." So I'm not telling them how I would do it necessarily. Well, I am sort of. I'm trying to say, "Some of you have thought of these things and I think that they're valid (ways) to present it." (3, March 25, 11)

The teacher used another writing question comparing two concepts later in the course. He was pleased to note that students had incorporated some of the ideas discussed.

Teachers found that students' written responses gave direct feedback on the effectiveness of instruction on

writing skills. They were pleased to see that students were responsive to this help.

Summary Student responses contained information relating to the effectiveness of:

1. general mathematics instruction;
2. different teaching techniques;
3. instruction on specific topics;
4. classroom communication; and
5. instruction on writing.

Student feedback on general mathematics instruction was largely disregarded by teachers who tended to see student suggestions as excuses for lack of success in mathematics. In some instances, however, teachers did realize that student suggestions might have relevance and, consequently, made changes in instruction.

Some student suggestions revealed a lack of confidence on the part of the student and a desire for more support from the teacher. Appropriate teacher attention to such responses might help to overcome some of this anxiety.

Although one teacher asked students for feedback on the effectiveness of a teaching technique, most awareness in this area came from inferences by teachers. Teachers compared previously-used teaching techniques to each other, to new ones contained in student responses, or to writing itself.

Student feedback on the effectiveness of instruction on

specific topics came from teachers' inferences about an entire class's understanding as revealed through writing. Some reteaching of topics was done. But often participants did not have time to go over topics again. Plans for revised instruction in future courses were sometimes forgotten.

Teachers became aware of students working together and of one student who had a visual difficulty. Potentially, such discoveries have implications for seating and organization of students. The quality of written responses was improved through instruction. Writing activities did provide teachers with the opportunity to become more aware of the effectiveness of their classroom instruction.

## Teachers' Awareness of Students' Attitudes and Feelings

Teachers comments revealing their awareness of students' attitudes and feelings fell into three categories. These related to students':

1. emotions;
2. attitudes to studying mathematics; and
3. conceptions of mathematics.

Emotions. Student responses showed the wide range of emotions felt in the mathematics classroom. Feelings varied from rage to enjoyment; from frustration to satisfaction. Students revealed their innermost feelings and teachers could not help but appreciate the glimpse into students' minds.

> Prompt: Math is like a mountain. You have to climb over it to get to the other side. Some parts are easy and some are hard. But if you keep going you will eventually get to the top. Write a paragraph telling me what you think mathematics is like. Use you own images.
>
> Student: Math is like water skiing, you have to get up out of the water onto the surface and be able to stand up. Then you hit some rough water and you almost fall down but you've got to keep trying to stand up. If you don't fall down it gives you an exhilarating feeling to know you made it around the lake without falling. (1, DF, Feb. 12, 3)
>
> Student: I think math is like music without a melody. The music is there, the notes are there, the dynamics are there (but I don't feel the depth). Math and music are both logical, i.e. they make sense, I like math to make sense so I can understand it. (1, NB, Feb. 12, 3)
>
> Teacher: Some of these would be neat to put up - as a quote on the wall. It's a neat question, a good one. I like to read what they've said - some other way of describing math. This is neat to read. (1, Feb. 12, 3)

Many students expressed less positive feelings toward mathematics. They created graphic images to show the frustration, fear, or anger they felt. For some teachers these pictures clarified what they saw of students in the classroom.

> Student: Math is like a John Deer tracter (sic) falling on your head. One is happy and unsuspecting until one day it squishes your brain. There is no way to get around it and it will always get you. (4, SS, Dec. 9, 4)
>
> Teacher: She's a good student actually. She's an honor student. She works hard, works really hard. She seems to be like a "nervous Nelly". That's the only phrase I can think of to describe her. She seems so tense a lot of the time and whenever she gets ... excited, she starts talking fast and short of breath and things like that. ... I guess she's waiting for the tractor. "One is happy and unsuspecting until one day it squishes your brain. There is no way to get around it and it will

> always get you." These are very insightful, aren't they? (They) tell a lot about (the students). (4, Dec. 9, 4)

Other teachers were surprised when students expressed resentment at the way in which they were treated in class. Students from non-academic courses, who had switched to Mathematics 30, felt that their different background should be recognized. The comment, below, does not ask for special treatment, but for acknowledgment of a different background in mathematics. The teacher looked thoughtful when she read the student's writing.

> Student: Math is like an endless line that keeps on going and going. The worst of it, is the endless part. The work is never ending stuff that I'll never ever use. Numbers and letters should not be mixed. A lot of my troubles are because the course is so far from Math 33. Math 30 students seems to be treated as if 33's never existed. (2, SM, Feb. 18, 5)

> Teacher: It's amazing they say what they do ... Actually, they have something to say. (2, Feb. 18, 5)

Some students were completely negative, even angry about mathematics. For the most part, participants were not sympathetic to these students.

> Student response: I'm not taking this course because I enjoy math. Personally I think Math sucks and school sucks, but I guess if I want to live half-decently in this corrupt world I have to take this course. (2, GC, Nov. 26, 1)

> Teacher: There's a very emotional one. I just have no patience with (him). He's not that interested. He doesn't like to show his work. He won't follow directions.

Such student responses reflected what teachers saw in these

students every day in class. It was undoubtedly depressing to experience this total rejection of mathematics and, hence, of mathematics teachers. As one teacher commented, what students wrote had great impact.

> Teacher: When you read something, it kind of hits home more than in class. When you're teaching, you can kind of ignore it. (3, Dec. 18)

Emotional responses were not limited to the content of student writing. Sometimes, the act of writing itself elicited students' feelings. Sharing with someone, even the teacher, provided emotional release.

> Prompt: Write your own personal math history. (2, Feb. 4, 1)
>
> Student: I didn't start school 'til 9. So I have miss (sic) the beginning of basic learning. Then I sort of jump in grade six and begins there. I was really bad at division and multiplication. But I work real hard and manage to pull through grade six. In grade seven, I still have the same problem plus fraction. I would spend 15-20 mins on one question. My parents couldn't help me because they were working night shifts.... (2, JN, Feb. 4, 1)
>
> Teacher: This... Vietnamese girl did not start school until 9 years old. She looks after her younger brothers and sisters. Mom and Dad work night shifts. ... She appeared in tears when she told me. (2, Feb. 4, 1)

Even the style and format of written responses told of students' emotions. One boy wrote each response with smaller and smaller print, gradually retreating to one corner of the page.

> Teacher: Isn't that tiny? Another unit and he'll feel very 'unfinishable'. (2, March 19, 10)

Eventually the student made no response at all. Soon after, he dropped the course. His decreasing confidence and interest in the course were clear from the way he wrote.

Teachers were quite sure that they had received information on students' emotions from writing activities. When challenged, one teacher was adamant that she had become more aware of students' feelings.

> Teacher 2: It gave me really good feedback. ... "I panic. I hate it. I've never liked it. I have always been good at it." ... You don't release feelings readily in a classroom as a rule.
>
> Teacher 1: But how much of their feelings are they going to really release to you as a teacher, even in that written work?
>
> Teacher 2: They do.
>
> Teacher 1: But, I mean, a real release of feelings? I kind of doubt it. I don't know. I sure wouldn't with ... any teacher I ever had.
>
> Teacher 2: Anger, some kids will show you anger.
>
> Teacher 1: In a round-about way they might.
>
> Teacher 2: (This boy expressed his feelings) the other day in a very direct way. ... "I don't like this. I don't like that. I don't like anything." (He) is angry at all kinds of things. But when he arrives in class he won't talk to anybody. He won't say anything to anybody. (2 and 1, April 14)

Teachers found that writing provided information that would not be obtained in other ways. Students would not speak up in front of their friends, but could write in privacy. They did not have to risk being embarrassed in public.

> Teacher: When they write about feelings ... some will respond in these conditions, but not verbally in full view of the class or even one-on-one. It becomes less prone to criticism/embarrassment. (2, questionnaire, 1)

Participants found that this emotional feedback made a real impact on their perceptions of students. It intensified and sometimes modified the impressions held, largely on the basis of class behaviour and marks. Writing offered teachers a way to get to know student. It also offered a way for teachers to do more than just transmit information.

> Teacher: If you don't ask (about students' feelings), they're just bodies there and we're just delivering information and trying to give help. ... Some don't know what happened to them. Some (write) personal stuff. Some (write) nothing from their life. ... I think it (helps) you at the (beginning of the course) to identify something with that name, with that face, a bit sooner. (2, Feb. 4, 1)

One teacher anticipated getting to know students better by using writing. The class that was referred to was scheduled to join the study in February.

> Teacher: It will be interesting to see the responses next semester. (It is a ) good class, but quiet. I'll probably appreciate them more if I see some writing. I don't get any interaction at all. They're just out there. I don't know them personally at all. They're just quiet in nature. I'm interested to kind of meet them through this. Some of them, I bet I haven't talked to yet. (3, Nov. 27)

Participants indicated that information on student feelings would necessitate changes in their teaching. Teachers hoped to change the way they related to students and perhaps the way students felt about mathematics.

> Teacher: (Will questions about feelings change how you teach?) In the sense that, ... I have a little bit more empathy for them. Because they've never liked it and they've always hated it and they've never done good. You'd kind of like to change that. So in that sense, yes. (3, Feb. 26, 7/8)

One teacher modified her mode of instruction based on emotional feedback from student writing.

> Teacher: I gave another writing question ... "When you say the word math, what do you think of?" Something like that. You could see where they were coming from. They needed some success, so I'm trying to structure them a little more. They did better on the last test. You could really see the undertone from that question. ... We get the emotional part of that one. (2, Nov. 26)

Thus, writing activities caused changes in teachers' awareness of student emotions and consequent changes in instruction.

Attitudes to studying mathematics. Students' writing did express a wide range of attitudes to studying mathematics. Some responses were very positive.

> Student: I am taking this course because I enjoy mathematical problems. Possibly pursue a career with the knowledge I hope to achieve honors and accomplish full understanding of the mathematics area. (2, SW, Nov. 26, 1)

Others were very negative . Such responses disturbed teachers greatly. Participants saw poor attitudes as a great handicap to students' success in mathematics.

> Prompt: Is Math different from other subjects? How? Why?

> Student: Yes, math is very different from other subjects. It requires much more thought and brain power from the "victum" (sic) and often leads to insanity. It is very difficult, and often times makes very little sense. It is not a steady process such as English and often contains many `easy' rules, that occupy half your time to figure out, or remember. It is not memorizable like social studies, and is not logical like physics. It just tends to be a pain and remains to be something that chances are you will never use again. (3, PC, Dec. 4, 4)

> Teacher: They don't want to bother. They don't think there's any value in math. They don't see that it's any use in real life situations. ... It's their attitude to math. It's their problem. They're dragging themselves down. (3, Dec. 4, 4)

Teachers sometimes found reading students' responses very depressing. It reinforced the negative attitudes they saw and experienced in students every day in class.

> Teacher: Reading 25 out of 30 times that students think math is stupid or not what they want to do, or whatever, isn't useful to me. (3, questionnaire, 1)

Other students failed to see any connection between work done in class and marks received.

> Student: I hope I can get a 70% or more in this course but too bad I only got 33% (average). I hope that the marking procedure are (sic) not that strict. (3, KC, Feb. 5, 1)

> Teacher: (He) wants 74% and he's getting 33. There's no indication he feels any responsibility for that 33 at all. It's just luck. He has no hope, but he doesn't know it. He has hope: that's the sad thing. ... He's hoping the test aren't marked so strict. That's the downfall of him. (3, Feb. 5, 1)

Teachers' awareness of this gap was enhanced by reading student responses. They even hoped that writing activities might help students to wake up, to realize what they needed

to do. Participants saw that students had to learn the lesson for themselves, telling them was not effective.

> Teacher: They are struggling because they don't think about what it takes to succeed in math. It's not the same from me. We talked about it in class. But some just need to analyze it for themselves. Most recognize what it takes, but there's a huge gap between what they recognize as the problem and what they're willing to do about it. Maybe a question like this might get some of them to stop and reflect on what it takes to learn math if they try and look at it a bit differently. (4, Dec. 9, 4)

Some students knew exactly what they had to do to succeed in mathematics. This was almost more depressing than when students did not. More than once students promised to work harder and do more studying. Teachers were disheartened to see how few actually did anything to change their work habits.

> Prompt: What kind of mark do you hope to get in this course? How does it compare to the mark you got on your last report card? What can you do to get the mark you want?
>
> Student: The mark I hope to get in this course is at least a 75%. I feel I should be able to get above 80%, however, On my last report card, I got a 62%. This mark is 13% to 23% lower than I want it. As soon as I finish writing my midterm exam, I knew that I had to spend more time studying. I plan to set aside an hour each night dedicated to both doing and studying the assigned homework and reviewing all the work we've done since the beginning of the year. I feel that by doing so I will be able to achieve the final mark I want. (3, VF, Feb. 5, 1)

Later in the course , the same student responded:

> Prompt: What studying have you done already (the test, tomorrow)?

> Student: In all honesty, I have not spent any time studying yet. It's always on my mind, but I never get around to doing any. (3, VF, Feb. 26, 7)
>
> Teacher: It's a little bit frustrating when you read the same thing all the time. They know what they're supposed to be doing, but they don't (do it). (3, April 22)

Other teachers made similar comments.

> Teacher: They'll do the same thing next time. I think once it's ingrained by this time, they don't change much. ... They don't (study harder). They just say that. They have no intention - these kids. Well, some of them do, but its a token like "I'm sorry". ... Just a good way to pass off what's happened. I find that they're superficial about, "I should have studied more." To me they say it over and over and over again, but they don't do anything. (2, Feb. 25, 6)

It is perhaps not surprising that students did not follow through with their promises. They appeared to be writing for the teacher (transactional writing), rather than for themselves (expressive writing). They wrote what they thought the teacher wanted to hear.

But some students wrote expressively, for themselves. This writing did make a difference. Students promised themselves a change in attitude, and sometimes this change materialized.

> Prompt: What kind of mark do you hope to get in this course? How does it compare to the mark you got on your last report card? What can you do to get the mark you want?
>
> Student: I was hoping to achieve a 70% or higher. My mark was fifty-two. I have to work in math and start studying every night. I will do this!!! HOMEWORK WILL improve my mark. (3, RR, Feb. 5, 2)

Three weeks later, another writing activity showed that the student was trying harder and was doing some work.

> Prompt: What studying have you done already ( for the test tomorrow)?
>
> Student: I've have (sic) read through each chapter and worked on the review questions of the old diploma exam questions. (3, RR, Feb. 27, 7)

The boy's marks had not improved very much, but his attitude and work habits had. His teacher was encouraged and a bit surprised by the change in attitude. Writing revealed a side of the student that had not been visible in class.

> Teacher: In a lot of cases, ... although they don't show it, (the students) would like to know how to do it. They would like to understand and do well in math. I think a lot of not showing it is who's around. (3, Feb. 27, 7)

Teachers found that becoming aware of this kind of information about students gave them more empathy for students, particularly for those who really had trouble with mathematics, but sincerely wanted to do well.

> Teacher: I have a lot of people in there who have never done well in math, have always hated it since grade whatever. (They) get really frustrated quickly because they can't do it. No one ever really explains it to them right. ... So I can appreciate where they're coming from. ... (Writing) gives me empathy for where those kids are coming from, which I think ... is really important. You lose that lots of times. ...And you find out that a lot of these kids genuinely would like to be able to do math, but they have never been able to do math. ..I think that as a teacher I have to remind myself of that from time to time. ... Some of these guys do want to learn. They just have had a really hard time with it. ... And I think this (writing) helps you (understand) that. (3, Feb, 14, 3)

Teachers perceived that writing activities provided information on students' attitudes to mathematics that could not have been gained through other means.

> Teacher: The other day, ... we did the one with the midterms. (I asked,) "How did you feel about the midterms? And explain what's been happening with your absences and lates." I got information I never would have, ... otherwise. They'll tell you things when they write it down that they wouldn't tell you face to face. Someone might hear or (there might be a) lack of opportunities (for doing so in class). (2, Feb. 4, 1)

Student writing did convey information about students' attitudes to studying mathematics. Teachers were surprised to find out about some of the students who had a good attitude, worked hard, and genuinely wanted to learn. Participants came to see that this kind of attitude was not necessarily connected with students' marks. Teachers developed more empathy with and, perhaps, more tolerance for students with good attitudes, but poor achievement. Teachers appreciated that trying is also important in mathematics.

Conceptions of mathematics. Writing activities revealed that students held a wide variety of conceptions about mathematics. Teachers became more aware of these as student writing highlighted facets of students' views.

One of the first insights that teachers had into students' views of mathematics was that some students felt that writing did not belong in a subject like mathematics. Expressing mathematical ideas in one's own words did not

match students' images of appropriate mathematical activity.

> Student: I feel that written types of questions should be left to the courses in which there is not a "right" or "wrong" answer. It is difficult for a student who for his/her entire life has been taught that math has "right" or "wrong" answer to accept a possible "median". (3, TT, April 12)

> Student: Personally, I have very little problem answering with words. I believe that I can clearly explain myself and my work. However, I think that math is about numbers and not about words. Mathematics is a sacred institution that must be protected from opinions that will destroy it. Math should be right or wrong, black or white. (Within reason of course, because if a certain number of decimals aren't asked for, 7.9214 and 7.921 are essentially the same number). (2, NP, April 7)

> Student: Math is like nothing else or any other subject. If you are born and you can do it then fine. Some people can do math and some people can't. So either your (sic) good at math or English, rarely the two people are good at (both). (1, DC, Feb. 12, 3)

> Student: Math is different from most subjects in that they deal with thoughts and ideas that can be expressed in words. Math is basically numbers. ... Math teaches you how numbers relate, ... (3, JW, Dec. 4, 4)

> Student: Math works with numbers, there's always a right answer, and other subjects involve either thinking creatively or synthesizing. Math is more rigid, less lenient. (3, MC, Dec. 4, 4)

These responses revealed that many students saw writing and mathematics as completely separate activities. To them, mathematics dealt with numbers, not thoughts and ideas. Language was not part of doing or learning mathematics. Writing implied accepting answers that may only be partly right. Mathematics required students to get the one right answer.

Teachers became aware that some students viewed writing and mathematics as unrelated activities.

> Teacher: Well, it's a really tough thing for the kids to get used to. ... But really it's just another way of expressing yourself and it's still math, but they ... see it as writing as opposed to math. (Students see them as being in) different compartments. (3, April 22)

While participants did not analyze these misconceptions, it appears clear that many students had what Borasi (1990) calls 'dysfunctional' mathematical beliefs. These students viewed the scope of mathematical activity as finding the correct answer to a problem. Appropriate mathematical activity was solving these problems with numbers, not exploring ideas. The personal judgments and values inherent in writing had no place in mathematical knowledge. Mathematics was not doing, but reciting.

All participants found this frustrating and a bit depressing. Teachers perceived that students wanted to passively absorb mathematics, not put any effort into learning it.

> Teacher: Some of these kids will be in real trouble if they have to think. It's not something they're used to doing. The accent is on using the formula. They don't want to understand. "Just give me the formula. Tell me how to do it." (4, Dec. 16, 5)

> Teacher: They don't know what they're doing. They don't know why (they do anything). They trust me. They don't have that second understanding. (They are) basically learning by memorizing. (3, Nov. 27, 1)

Teachers' perceptions were reinforced by students' responses to content prompts. Everything students wrote had the potential to reveal their conceptions of mathematics.

> Prompt: Explain in your words why $\tan(x) = \frac{\sin(x)}{\cos(x)}$. You may use a diagram to support your explanation.
>
> Student: It's a law. It's a mathematical property. You can't explain why $\tan(x) = \frac{\sin(x)}{\cos(x)}$. It is a mathematical property that is memorized. It is a fact.
>
> Teacher: For (him), it's a mathematical property that's memorized. He can't explain it. ...Some of these kids can't think. (It's a) mathematical property that can only be memorized. (1, March 25, 5)

Not all students shared these views, however. Some had much wider conceptions which again were revealed in their writing about writing in mathematics and in their writing about mathematical content.

> Student: Dear friend: Next year, in math 30, you will be required to do a lot more written questions. Don't be afraid, for they are not hard. I find they are a better indicator of whether or not you understand the concepts. If you understand the concepts you will have no problem doing them. I believe they should be a part of mathematics as well as being on exams, for it tests your understanding and not just your knowledge. (3, MS, April 12)
>
> Student: I feel that writing out an answer (and therefore thinking about it) helps me to better understand what I'm doing. I feel that writing out an answer requires me to analyze what I'm doing and therefore think instead of simply jotting down the appropriate formula and substituting. (3, JW, April 12)
>
> Student: If you are confused about a subject ... I think writing about it in your own words will help you learn and understand the concept introduced. (3, RR, April 13)

> Student: Maybe it won't be hard to answer a math question by using words, and I am uncertain of it because I did not do that kind of questions before. But it should be part of math because not everything in math can be expressed in math symbols and numbers. (2, GL, April 7)

Teachers perceived that such students accepted writing as a legitimate mathematical activity, useful in coming to understand mathematical concepts and in demonstrating this understanding to others.

> Teacher: The kids, I think, ... (said), "By writing it down, you know whether you know it or not." (2, April 14)

> Teacher: I really found that some students like this type of questioning in math. ... It does bring to light there's much more to math than just being able to do simple things and recognize things. You have to be able to articulate what you're talking about as well, (to) communicate. And I think a lot of students have recognized they fall far short. ... They're not objecting to (writing) at all. They seem to recognize this is a valuable experience, a valuable activity. (4, Dec. 2, 2)

Other students enjoyed doing mathematics for its beauty.

> Prompt: What is the most interesting thing you learned in Trigonometry?

> Student: The most interesting thing I learned in trigonometry is to draw beautiful curves of trigonometric functions. Those curves look so smooth and when I draw the curves of trigonometric functions, I know this world is doing the same thing - repeating itself. (2, GL, March 26, 14)

> Teacher: "Beautiful curves - so smooth" (It's) too bad we didn't get to the point where we add (trigonometric functions). Oh, he would have liked that. (2, March 26, 14)

Others enjoyed mathematics for the challenge it offered.

> Student: I enjoyed proofs because when you finally proved that one-side equalled the other side, it was a great feeling. I also found it challenging, and every once in a while I enjoy a good challenge. ... (2, SB, March 26, 14)

> Student: The fact that I couldn't grasp the concept of identities for the longest time, but when I did, it became the most reassuring and interesting. (2, DB, March 26, 14)

Several students appreciated the networks of concepts in mathematics and enjoyed putting them together as they built their own understanding.

> Student: The most interesting thing I've learned in Trigonometry is the Unit circle. The Unit circle provides the ability to understand trig graph, sum and difference equation and so on. The more trig you learn the more connections you see from problem to problem. (2, TS, March 26, 14)

> Student: (The most interesting thing I learned in Trigonometry is) that you do not need numbers to solve equations. Everything I have learned in all my years of math ties into this one unit. (2, RK, March 26, 14)

Teachers appreciated these more enlightened views even if they were a bit startled to find them in students.

> Teacher: I was surprised that identities were such a priority. ... I suppose it's like a puzzle - try this, try that. ... And sometimes they'd get so far and then they'd have to be creative again. ... I enjoy doing them, but whether that's a factor or not I have no idea. ... I did say to them, "The ones I really hate are where the left (side of the identity) is really similar to the right (side) except one is $1 + \cos(x)$ and the other is $1 - \cos(x)$." ... I remember there was one question that took me the longest time. ... It doesn't always come bang, bang, bang. ... It could be the fact that, "Gee, she's putting it together." It's hard to say. (2, March 26, 14)

In this last statement, there is a recognition that teachers' own conceptions of mathematics may influence their

students' views. This particular teacher had previously shared her metaphor of 'mathematics as construction' with her students.

> Teacher: Sometimes I tell the kids it's like a carpenter and these are your tools and if you get a poor foundation your house crumbles. (2, Feb. 18, 5)

One participant thought that exposure to writing activities might help to change students' conceptions of mathematics.

> Teacher: (Writing is) not a right/wrong thing, so (the students) don't get this impression of "What is the right answer?" ... There's an underlying problem there. ... "How do you get this right answer?" as opposed to "I don't understand how to do the question." ... This "I want to get the right answer. How do you get this answer?" really has nothing to do with learning mathematics. (3, April 22)

Teachers thought that information about students' conceptions of mathematics was valuable in giving them new insights into their students.

> Teacher: I think that writing is interesting because at the same time they were saying something that was somewhat creative. ... They are also saying ... what they think math is like - you get it or you don't. Like there's no happy medium, that's it. And so that tells a person something about the kid. (1, April 14)

For one participant, writing activities did highlight differences in students' views of the nature of mathematical activity.

> Prompt: Is math different from other subjects? How? Why? (3, Dec. 4)

> Student (Larissa): Math is something you have to learn how to do and not memorize how to do the problem. Math is different because instead of just knowing facts like in social you need to now (know, sic) how to apply your knowledge. (3, LS, Dec. 4, 4)

> Student (Kevin): With so many different formulas, you have to make sure you have the right one to answer the problem. There is a lot more memorization and repetition in math. It is , also, more precise than any other subject. (3, KC, Dec. 4, 4)

> Teacher: (Larissa is) not a good student, but she's one of the few that wants to learn and I get a good feeling about how she's approaching things ... when she comes up and asks me a question. Kevin will come up and say, "How did you get that (answer)?" She'll say, "How do you do this question?" (It's) completely different and it has nothing to do with being a good student. ... It has nothing to do with whether they can do math. He's trying to memorize. If he wants to know how to do a question, he wants to know how they got 32, not what's going on there. ... (He doesn't) want to understand the process. (3, Dec. 4, 4)

Writing activities enhanced teachers awareness of students' conceptions of mathematics. Participants felt such insight might lead to long-term changes in instruction.

> Teacher: If you can see what kind of conceptions or misconceptions kids have about what mathematics is all about, it seems to me that kind of information can't help but mold one's own approach in the classroom. This is important feedback. (4, Dec, 16)

Summary. Writing activities did provide teachers with the opportunity to become more aware of students' attitudes and feelings. In particular, student responses contained information on students':

1. emotions;
2. attitudes to studying mathematics; and
3. conceptions of mathematics.

Written responses conveyed a wide range of students' emotions to teachers. While some students found mathematics an enjoyable and rewarding experience, many more found it frustrating, frightening, or infuriating. Teachers found these negative emotions disturbing, but questions were not asked about what had caused students to feel this way, nor were there any concrete attempts made to help them.

Teachers rejected some very angry student responses outright, referring to the students' behaviours in the classroom. It would appear that the teachers, in turn, felt rejected. Writing did serve to open channels of communication in the classroom, but not wide enough for some students.

Written responses also made teachers more aware of students' attitudes to studying mathematics. As these were largely negative, participants found that being exposed to these attitudes in class and then again in writing was depressing. A few students, however, surprised teachers by revealing positive attitudes and a genuine desire to succeed in mathematics.

When students wrote to the teacher with transactional responses, intended "to communicate to an audience, to inform, persuade or instruct them" (Britton et al., 1975, p. 160), no changes in work habits resulted. Teachers were disappointed again and again. However, when a student

responded with expressive writing, exploring "what he or she thinks, feels or knows" (Britton et al., 1975, p. 90), promises had more chance of being kept.

It would appear that teachers' and students' views of writing are important here. These determine both the mode of writing and the amount of reflection done by students in writing. Expressive writing may play an important part in trying to change students' attitudes to studying mathematics.

Student writing revealed that many students hold 'dysfunctional' conceptions of mathematics. Teachers perceived these beliefs and felt that they were an important factor in students' success or lack of success in mathematics. Other students held more sophisticated views of mathematics.

Teachers' own conceptions of mathematics affected what they saw in student responses. They recognized that their views might influence students and that students' views, in turn, might affect theirs. The interactions between teachers' and students' conceptions of mathematics certainly have an affect on both learning and teaching. Teachers need to be more familiar with common misconceptions of mathematics, and of ways in which these can be modified through mathematics instruction. Writing may be instrumental in assisting teachers to do this.

Writing activities did reveal much about students' attitudes and feelings to teachers. As teachers became aware of these emotions, their understanding and empathy for student increased. It is not clear that much change in teachers' relationships with students or in their teaching would result, but teachers gave some indication of seeing the need for change. Some moves were made in this direction.

Teachers' Awareness of Students' Understanding of Mathematical Content

Writing activities did help the teachers to become aware of students' understanding of mathematical content. Students were asked to write about concepts and processes presented in Mathematics 30. Written responses revealed not only what students understood, but how they understood, as well. Writing activities contained information about five different areas related to students':

1. background knowledge;
2. knowledge of Mathematics 30 concepts and processes;
3. use of metacognitive strategies;
4. use of mathematical language; and
5. modes of understanding mathematics.

Student responses assisted teachers to become more aware of students' understanding in each of these areas.

Background knowledge. Written responses revealed gaps in students' knowledge from previous mathematics courses in junior and senior high school. Students were able to identify their own weaknesses in explicit statements responding to content, affective, and instructional prompts.

> Prompt: What is the hardest thing about solving problems involving permutations and combinations?
>
> Student: The hardest thing about these problems is that you can't just use one formula to solve them. ... The hardest problems to me are those in which you have a number of different possibilities and when I have to be careful when I use nCr and when I forget whether to multiply or add all the possible combinations. (4, FB, Dec. 16, 5) (emphasis added)

Other students identified coordinate geometry, basic arithmetic, and algebraic skills as problem areas. Teachers did acknowledge what students said about their background, but none of the information was new. All difficulties identified by students were areas which typically cause difficulty for students.

Writing served, not so much to enhance teachers' awareness of student difficulties, but to identify students who were capable of diagnosing their own weaknesses. Writing activities provided an opportunity for students to reflect on their mathematical skills. Students who acknowledge their own weaknesses might be more disposed to accept remedial help.

On several occasions, teachers gained new information from student writing. Students, who recently transferred into the school from out of province, identified gaps in their prerequisite knowledge.

> Prompt: Which concepts (on the test) didn't you know? What could you have done differently to prepare for the test?
>
> Student: I am a little nervous about the test, because I am not fully understand with yes-no survey, especially the "90% box plot". It is brand new to me. ... The yes-no survey is the hardest part to me. (2, WW, Dec. 3, 3)
>
> Teacher: This girl just transferred in from somewhere. ... She may not have run across box plots before. (2, Dec. 3, 3)

Teachers' awareness of students' background knowledge in these situations was enhanced. There was, however, no evidence of participants giving help to individual students with difficulties. Teachers found the pace of instruction too fast to permit remediation.

> Teacher: You should be able to (do remedial work). But it's a function of time. You can sometimes comment to the kid as you go by. (2, Feb. 25)

Mathematics 30 concepts and processes. Responses to content writing prompts revealed the degree of students' understandings of Mathematics 30 concepts and processes. On several occasions, participants found that students had a good understanding of mathematical content.

> Prompt: Imagine that you are writing to a friend who really wants to know the difference between a geometric sequence and an arithmetic sequence. In your own words, explain as many differences to him or her as you can.

> Teacher: I think most students had a pretty good idea what an arithmetic sequence is. They had a pretty sound understanding what a geometric sequence was. They seemed to understand that with arithmetic sequences the key characteristic as you go from term to term is you're adding and with geometric (sequences) you multiply. My impression is that most students understand that. (4, Dec. 2, 2)

With other writing activities, teachers were able to see that their class as a group lacked understanding.

> Prompt: Imagine that you are writing to a friend who really wants to know what a trig identity is. Explain this to him or her using your own words.
>
> Teacher: They don't really know what ... it is. Very few of them used the fact that it was a true statement, although I must have used that (phrase) a million times. ... I was thinking that they could use the words "true statement" - those two words. I would have been happy if they ... said "true statement" and (that it) had something to do with trig. (3, Nov. 27, 1)

General lack of understanding reflected on the effectiveness of instruction. Teachers acknowledged that the topic might have to be reviewed. While some teachers retaught lessons in response to this feedback from students, most carried on with new topics. Even when remediation was planned, teachers found they didn't have time to fit it in.

> Teacher: I find these kids can't spend enough time on the concepts. . . . I feel sorry sometimes. I find I'm doing something because I have to get on. I just don't have the time (to go over it again) (2, Dec. 9, 6)

Student writing was also useful for diagnosing individual students' difficulties.

> Prompt: What would the graph of an arithmetic sequence look like? Explain why using your own words.
> What would the graph of a geometric sequence look like? Explain why using your own words.

> Student: The graph of an arithmetic sequence would be linear with a relatively small slope. Because an arithmetic sequence involves adding a common number to all the terms, the angle of the graphed line would be quite small. Addition only slightly increases the term. The graph of a geometric sequence would also be linear, but the slope and the angle of the line would be much larger than that of the graph of an arithmetic sequence. A geometric sequence involves multiplying a common number to all terms and therefore the size of each term increases or decreases more than if it were addition. (MW) (emphasis added) (3, MW, March 18, 9)

> Teacher: I don't know what's up with (this girl). I don't really know what's going on. ... She must be busy with other things because she's not concentrating on math. She really has no conception what's going on with a geometric (sequence) ... at least as it relates to a graph. (3, March 18, 9)

Teachers related perceptions gained from written responses to previously-gained knowledge of students. Participants were conscious of interpreting student responses.

> Teacher: As a teacher you most likely have to read "between the lines" as to what they are trying to say. (2, questionnaire, 1)

Written responses did have an impact on teachers' perceptions of students. One teacher's comments revealed that her estimation of a student changed gradually in reaction to his writing.

> November 26: Nothing really there at all. He's a Math 33. (2, Nov. 26, 1)

> December 3: Well, that's pretty good. It shows some understanding. (2, Dec. 3, 2)

> December 9: His responses are pretty simple, ... not bad for a 33. When I see now everything he's done (in writing), you can see there's something there as compared to (some of the other students). (2, Dec. 9, 6)

> December 17: There are some surprises (from) kids who don't respond in class. You can pick it up on paper. They might say it on paper, but they won't say it out loud. Like some of the oriental kids - and (this boy). ... Every time, he's pretty close. He doesn't elaborate, but to me, he's not vague. He's (right) on there, but his vocabulary isn't strong. (2, Dec. 17, 7)

Writing provided the opportunity for this student to show the teacher his understanding of mathematical content, where regular classroom interactions had not.

Participants agreed that writing revealed the extent of student understanding.

> Teacher: (I get) feedback on how they conceptualized the concept or lesson. I can get into their minds better. (1, questionnaire, 1)

> Teacher: Content area questions give immediate feedback on the kinds of things kids are struggling with. (such questions) need to be given. ... They give immediate feedback on what's happening in the classroom. (4, Dec. 16)

> Teacher: It's an eye-opener to read about students' conceptions and misconceptions of concepts. (4, questionnaire, 1)

> Teacher: You can see whether the class as a whole is on track. ... I can learn who is having difficulties. (2, questionnaire, 2)

> Teacher: You can tell if they know what they're talking about or not. And if they don't understand it enough to write it down, then they obviously don't understand it enough to remember it a week from now or two weeks from now. And they can probably do the questions that day, but that really has nothing to do with whether or not they know how to do the mathematics. (3, April 22)

Teachers believed that students recognized that written responses revealed their understanding.

> Teacher: The big thing about writing is ... that if you don't know what ... you're talking about, you can't (bluff) your way through. I have no sense that my guys think they can (bluff) their way through.(3, March 2)

While information gained was useful, it could be depressing.

> Teacher: It doesn't do (me) much good in terms of feeling like the class is working and following ... (if only) 5 or 6 can do (the question). ... It's depressing to ask a question like that and everyone watches you walk around the room. They have no idea what you're even talking about. ... So (writing is) good, but it can get frustrating. (3, Dec. 18)

Although the information in students' responses sometimes frustrated teachers, it did enhance their awareness of students' understanding of Mathematics 30 content.

Use of metacognitive strategies. Metacognitive strategies determine students' awareness and control of their learning. Involved are the ability to reflect on one's actions, the ability to bring into play strategies for dealing with novel situations, and the ability to oversee and to control one's use of these strategies. The possession of metacognitive skills is a key factor in successful mathematics learning (Campione, Brown & Connell, 1989).

Writing activities were able to enhance teachers' awareness of their students' use of metacognitive strategies. Many students proved to be unable to analyze or identify strategies for doing problems.

> Prompt: What is the hardest thing we studied in the unit on Trigonometry? Write down a strategy for doing this.
>
> Student: identities. I have no strategy. I am still confused. (2, AC, March 19, 11)
>
> Student: I find that identities to be very difficult. I guess one strategy would be to memorize some formulas, Pythagorus and do some examples daily. (2, MP, March 19, 11)
>
> Teacher: They don't have strategies. ... When they say "strategies", it's always ... "How would you improve your mark? Study harder. Do more questions." ... I said, "Well, that's a general thing. We want to get something specific." ... I've been trying to show them that when you're doing something you don't just do the mechanics, you have to look at what happens. ... I've tried in a number of ways this week with a number of situations to say, "As you're doing something, do you not pick up on little things?" And I don't think they (do). (2, March 19, 11)

Other students were capable of analyzing mathematical problems.

> Prompt: Using your own words, write down a set of steps for solving trigonometric equations.
>
> Student: $\cos^2 x + \cos x = 1$
> - move right hand equation to left to make it zero
> - try to factor
> - if factor doesn't work use quadratic formula
> - find 2 solutions
> - use unit circle to find the rest of the answer
>
> (2, JN, March 26, 13)
>
> Teacher: She's not bad. For (her), it's actually very good. She actually uses a specific example, which is interesting. (2, March 26, 13)

This student not only was able to outline a method of solution for the equation, but demonstrated a metacognitive strategy by thinking about a specific equation rather than

trigonometric equations in general.

Writing activities pointed out differences between individuals and differences between groups of students. Teachers thought that maturity was a factor in metacognition.

> Teacher: To me there's a difference between the two (grade 10 and grade 12). Well, (the grade 12's) were basically taking the major idea of the day and (trying) to write down the basic point. They could do that. The 10's ... don't know where to begin. (2, March 19, J/10)

Reflecting on their own experiences as learners, teachers felt that these metacognitive skills were important tools for learning. They hoped to assist students to learn how to use metacognitive strategies in their own learning of mathematics. Writing was one tool for doing this.

> Teacher: Examples of ways of learning mathematics can be learned by (students). By class discussion (of writing activities, I) can show or demonstrate a possible way to achieve this. (2, questionnaire, 2)

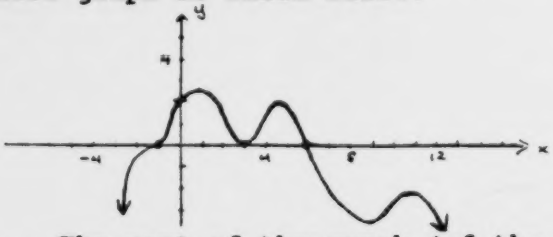
> Teacher: (This might be a start?) I think so! I think it's good that they actually look back. ... When you're writing a test, you should be able to get a feel for what's going to be on it, from the way it is taught. I think that's a really good skill to pick up and I think it helps the top student. They might do this already - think about what's going to be on the test, but if you get a top student who doesn't, then it's a good thing they could pick up quite easily. ... Now this (writing activity) is good. ... But you know, you're not going to catch all of them, not all of them are going to say, "Gee, I really want to learn this." You're only going to catch the motivated ones. But this is good, because then they might actually think next time, "Gee, what kind of questions are going to be on that test", as opposed to,"Gee, I have so much to study." (3, Dec. 18)

Some teachers followed writing activities with discussion in class to help students develop their metacognitive skills.

Mastery of mathematical language. Written responses highlighted students' use of mathematical language. Teachers were made aware of weaknesses in individual student's writing and vocabulary. Students often had difficulty interpreting questions. Often they did not notice key words in writing prompts.

> Prompt: For the sequence -2, 6, -18, ... explain how you could predict, without doing any calculation, that 4374 is an even-numbered term.
>
> Student: It would be an even number because the common ratio is -3. Since the first term is a negative even number, then the next will be a positive even number. (odd x even) = even So 4374 will be a term. The pattern of odd x even = even will continue until you get 4374. (4, RJ, Dec. 2, 3)
>
> Teacher: Students are confusing an even number with an even-numbered term. (4, Dec. 2, 3)

Poor use of mathematical terms also limited students' ability to respond to writing prompts. Terms from one branch of mathematics were often applied to questions in another.

> Prompt: Explain to a friend who has not taken Math 30 yet what you can say about the zeros of the polynomial function whose graph is shown below.
>
> 
>
> Student: ... The part of the graph (of the polynomial function) which crosses over the x-axis illustrates <u>two distinct zeros</u> and the area of the graph where it touches the x-axis and then changes direction illustrates <u>one equal zero</u>. (2,SB,Feb. 11, 2) (emphasis added)

> Teacher: She's starting to confuse crossing with two distinct zeros. She's confusing quadratic function terminology. ... The class is really unsure of quadratics. We spent part of one class (reviewing quadratics). It wasn't enough. They need a good solid class (on it). (2, Feb. 11, 2)

Teachers found it hard to get students to use the correct mathematical terminology. Students tended to revert to informal language no matter how often the terms had been used in class. Non-mathematical terms were mixed with mathematical ones.

> Prompt: Write out your own definition of an infinite arithmetic sequence.
>
> Student: Infinite arithmetic sequence - is a <u>series</u> of numbers that follow the formula $t_n = a + (n-1)d$ and has no number which ends the sequence. It is infinite. (2, TC, Dec. 17, 7) (emphasis added)
>
> Teacher: That's what he's doing. ... He's using "series" generically (instead of mathematically.) (2, Dec. 17, 7)

In spite of these difficulties, participants perceived that many students were able to express their mathematical ideas using words. Written responses might not be polished, but they contained good ideas.

> Teacher: I was quite surprised at how well they did say what they had to say. (2, Feb. 11, 2)

Both students and teachers appreciated that both the terms and style of writing in mathematics might be different than those in other subjects. They recognized that in some respects mathematics is a different language.

> Prompt: Write your own personal math history.
>
> Student: My math history is a history of problems. ... I only have interest in financial math, all other math is like a different language that I can't understand. (2, RW, Feb. 4, 1))
>
> Teacher: Math is a different language! (2, Feb. 4, 1)

Writing activities revealed the importance of mathematical language. This added a new emphasis to participants' mathematics teaching. Writing might encourage students to use mathematical vocabulary more often.

> Teacher: I think that (writing) really forces someone to rethink what they are doing and to use the vocabulary. I think it takes quite a while to change that, in order to be more verbal. Kids are not used to it. (Dec. 17)

Writing activities were seen to be an important means of improving students' ability to use mathematical language.

Modes of understanding mathematics. Written responses shed light on students' modes of understanding. Responses revealed wide variations in the sophistication of students' thinking about problems.

> Prompt: You are offered a job that pays $1 per hour the first week, $2 the second week, etc. Would you rather that your wages increased arithmetically or geometrically? Explain why.

In the first response below, the student calculated terms in each sequence before coming to a conclusion. She is dependent on the context of the question.

> Student: If my wages increased arithmetically then after 10 weeks I would be making 10 dollars an hour.
>
> 1, 2, 3, 4,... $t_n = a + (n-1)d$
>
> $t_{10} = 1 + (10-1)(1)$
>
> $= 1 + 9$
>
> $= 10$
>
> If my wages increased geometrically, then after 10 weeks I would be making 512 dollars an hour.
>
> 1, 2, 4, 8, ... $t_n = ar^{n-1}$
>
> $t_{10} = (1)(2)^{10-1}$
>
> $= (1)(512)$
>
> $= 512$
>
> Based on these calculations I would rather my wages were increased geometrically, because I would make much more money. (1, WH, Nov. 29, 3)

The teacher thought this was one of the best responses. She decided to photocopy the solution for other students in the class to see.

Other students immediately used formulae in their responses. One calculated the tenth term in each sequence; another the sum of the first five terms. Such students were still dependent on the context of the problem, but less so than students who needed to list terms. Using sums of terms, in particular, indicates a more advanced level of reasoning. The teacher, however, preferred students to refer to terms in their solutions.

> Teacher: Some kids are trying to take sums of those (terms) which is a pretty silly thing to do. ... They should be looking at terms. (1, Dec. 4, 3)

Other students were even less dependent on the original sequences. One reached a conclusion and then added an example to support her reasoning.

> Student: I would rather that my wages increase geometrically since each week my wages would increase more than if my wages were increased arithmetically. This would happen because to increase geometrically each term (starting from the first term) is multiplied by the same number or ratio which generally increases the numbers to follow the first term faster.
> geometric 1(x2), 2(x2), 4(x2), 8, ...
> arithmetic 1(+1), 2(+1), 3(+1), 4, ...
> The geometric 4th term is double that of the arithmetic. (1, TB, Nov. 29, 3)

The teacher was aware that this showed a different level of understanding than the preceding students.

> Teacher: So this actually indicates that ... she understood what was going on before she had to use the example. (1, Nov. 29, 3)

A few students were completely independent of the problem situation. They were able to think about geometric and arithmetic sequences in general, without referring to the specific problem situation.

> Student: Geometrically. Because the numbers that go down the series increases (sic) tremendously where as the numbers that go down the arithmetic series increases uniformally (sic). (1, EL, Nov. 29, 3)

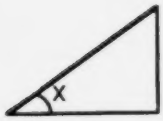
The teacher commented on the lack of reference to the sequence.

> Teacher: I would think that you do an example first, test it with some numbers, find out what happens, and then do a conclusion based on the example. ... I always think examples are good. (1, Nov. 29, 3)

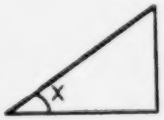
Written responses showed evidence of the process of students' understanding. Some students "folded back" to a previously learned, more concrete problem.

Prompt: Explain in your own words why $\tan(x) = \frac{\sin(x)}{\cos(x)}$.
You may use a diagram to support your explanation.

Student: tan x can be replaced with sinx/cosx. The sin x is the length of the opposite side. The cos x is the length of the adjacent side.
example:




$$\tan x = \tan 30 = 0.5773$$
$$\sin x = \sin 30 = 0.5$$
$$\cos x = \cos 30 = 0.8660$$
$$\tan 30 = \frac{\sin 30}{\cos 30}$$
$$0.5773 = \frac{0.5}{0.8660}$$
$$0.5773 = 0.5773$$
$$\text{LHS} = \text{RHS}$$




(2, MM, March 19, 10)

Teacher: He's trying to justify by picking an angle and giving a specific instance - like a verification. (2, March 19, 10)

The teacher recognized that the student's use of an example indicated a lack of understanding. The student was unable to relate to trigonometric functions in general and, thus, lacked a basic understanding of what trigonometric identities are.

Another student appeared to diagnose her own problem in understanding mathematics.

Prompt: Is math different from other subjects? How? Why?

Student: You cannot calculate anything out in relation to things we know. (eg amounts of something (fertilizer) used for something else (farmer's field)) You always end up with number for answers that don't SEEM to have any significance except for the fact that they are the answers. (3, GS, Dec. 4, 4)

> Teacher: She's missed that middle area. She needs to go back to real things. (She) can't connect (what she's learning) to math. She's reached her limit. She can't justify what she's doing to herself. Not that she doesn't want to learn. (But she needs to) at least pick up the ability to accept what we're doing. (3, Dec. 4, 4)

The teacher recognized that this student could not connect or relate different topics in mathematics. Knowledge of one concept was disconnected from other mathematical learning. Thus, the student needed to go back to concrete materials with every new topic studied, rather than being able to build on existing mathematical knowledge.

Teachers became aware of the extent to which students depended on formulae, memorization, and outside authority for justification of mathematics.

> Prompt: Explain in your own words why $\tan(x) = \frac{\sin(x)}{\cos(x)}$. You may use a diagram to support your explanation.
>
> Student:
>
> | | | |
> |---|---|---|
> | tanx | $\frac{\sin x}{\cos x}$ | tan x can be transformed into $\frac{\sin x}{\cos x}$ because they are mathematically equal. $\frac{\sin x}{\cos x}$ is found through tanx using the quotient identity |
> | $\frac{\sin x}{\cos x}$ | quotient identity | |
> | LHS = RHS | | |
>
> (2, KV, March 19, 10)

> Teacher: (She's) just treating it as an identity - because it says so. (2, March 19, 10)

> Teacher: They don't know what they're doing. They don't know why. They don't really know what an identity is. They trust me in that this is how we prove them and so they can probably prove them. But they don't really know what they're doing. They don't have that second

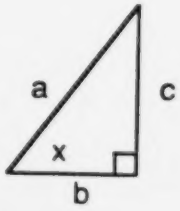
> understanding. They understand the basics of proving (identities), but that's it. They don't know what we're actually doing. They're basically learning by memorization. They don't really understand what we're doing. (3, Nov. 27, 1)

Participants became aware that other students reinforced previously learned concepts and constructed new understandings through writing. One student responded to the prompt, above, using a different approach than that used by his teacher or by the textbook.

Student:

$$\sin x = \frac{c}{a}$$

$$\cos x = \frac{b}{a}$$




$$\frac{\sin x}{\cos x} = \frac{\frac{c}{a}}{\frac{b}{a}} = \frac{c}{b}$$

$$\text{and } \tan x = \frac{c}{b} = \frac{\sin x}{\cos x}$$

(2, GL, March 19, 10)

Written responses illuminated students' understanding of mathematical structures. Some students were shown to be using an operational mode of understanding, thinking of sequences and series as processes based on operations.

> Student: a geometric sequence is a curved graph because when a number is squared the result accelerates. (2, RG, Dec. 17, 8) (emphasis added)

> Student: A finite sequence has an end.... The series comes to a halt. ...(2, SW, Dec. 17, 8) (emphasis added)

Other students thought on a structural level, conceiving of sequences and series as entities in themselves. The language used in their responses showed this.

> Student: A finite geometric series is an equation involving exponents that has a last term. (1,SD, Dec. 17,8) (emphasis added)

Teachers did not comment on the difference in these responses, although they did remark on the original terms and images used.

> Teacher: I've never used these words. (2, Dec. 17, 8)

Writing activities revealed great variation in students' modes of understanding. Teachers did become aware of some of these differences, but their opportunities to do so were limited. Samples of students' work can only be static snapshots of students' understanding. As such, they give teachers only a limited picture of students' understanding. A program of writing activities might give more complete and dynamic information.

Summary. Writing activities did provide teachers with the opportunity to become more aware of students' understanding of mathematical content. Teachers were able to get new insights into:

1. students' background knowledge;
2. knowledge of Mathematics 30 concepts and processes;
3. use of metacognitive strategies;
4. use of mathematical language; and

5. modes of understanding mathematics.

Written responses confirmed teachers' awareness of weaknesses in students' backgrounds. Some students, new to the school, were able to indicate topics missed in previous courses. This information added to teachers' understanding of students, but no remediation was done in response.

Student writing did assist teachers to diagnose general areas of difficulty and understanding of mathematical concepts and processes. While all participants felt pressured by time, material was occasionally reviewed in response to this information. Individual student's problems were also identified. Teachers often related these areas of difficulty to their previous knowledge of students. Individuals' strengths were also revealed and, on one occasion, changed a teacher's opinions of a student's ability.

Writing responses enhanced teachers' awareness of the use of metacognitive strategies in their classrooms. Generally, students were found to be lacking in this area. Teachers found writing activities useful in trying to develop students' metacognitive skills.

Students' use or misuse of mathematical vocabulary was clear from their written responses. Inappropriate use of mathematical and non-mathematical terms caused students to have difficulty interpreting and responding to writing

prompts. Teachers felt that writing was an important tool for helping students to develop their use of mathematical language.

Participants became more aware of students' modes of mathematical understanding. Writing responses revealed a wide range in students' levels of thinking about mathematics. The extent to which students depended on context, memorization, and operational processes was shown. Teachers became more aware of some of these differences

Conclusion

Writing activities provided information to teachers on:

1. the effectiveness of classroom instruction;
2. students' attitudes and feelings; and
3. students' understanding of mathematical concepts and processes.

Comments made in interviews and on questionnaires demonstrated teachers' awareness of this feedback. Teachers became more empathetic toward students. Some students revealed greater understanding of mathematical content in their written responses than previously perceived by teachers. While some instructional changes took place, participants felt they lacked the time to review or provide individual remediation. Some planned modifications to instruction were forgotten.

Writing activities did offer a means of communication between students and teachers. Students' voices were a rich source of information. Teachers listened and learned from them.

# Chapter V - Teachers' Reflections

## Introduction

This chapter addresses the fourth question explored in the study: Did writing activities provide teachers with the opportunity to reflect on their conception of mathematics and mathematics teaching? Evidence of such reflection is, by nature, less obvious than evidence showing that writing helped teachers to become aware of the effectiveness of instruction, students' attitudes, or students' understanding of mathematical concepts.

Reflecting on one's conception of mathematics or one's philosophy of teaching is not likely to result in immediate, readily observable changes. Modifications are more likely to be subtle and long-term.

It is difficult, therefore, to determine whether teachers did reflect on their conceptions of mathematics and teaching. In spite of this, ways in which teachers use any instructional technique are determined by their conceptions of mathematics and teaching. The uses made of writing activities in mathematics classrooms should reveal something of the attitudes and philosophies of the teachers involved.

All teachers involved in this study found writing activities to be a useful addition to their instruction of mathematics. Their reasons for coming to this conclusion and

the ways in which they implemented programs of writing differed greatly, reflecting their different philosophies of mathematics and teaching.

Changes in the ways in which teachers incorporated writing activities in their teaching were undoubtedly due, in part, to increased experience with the technique. But changes in participants' conceptions of mathematics and mathematics teaching may also be reflected.

This chapter, therefore, explores participants' programs of writing activities and the changes that took place. Three teachers, Mary, Ian, and Katherine, took part in both phases of the study. The fourth teacher, Gordon, participated fully in the first phase of the study. He withdrew from the study when assigned a student teacher during phase two. Data relating to the three teachers who participated throughout the entire study are reported here. These data were obtained from observations made by the researcher and comments in teacher interviews.

## Katherine

Background. Katherine had been teaching mathematics for nine and a half years. After spending three years in junior high schools, she moved to the high school level. The 1991/92 school year was Katherine's first at her present location. At university, Katherine received a bachelor's

degree in mathematics followed by a bachelor's degree majoring in mathematics education. During the study, she taught Mathematics 30 for the third time, although, as with all the teachers in the study, it was her first time under the latest curriculum changes.

Katherine usually used a lecture mode of instruction with her students. She greatly enjoyed teaching academic students. Katherine did not have her own classroom, but moved between two rooms throughout the day. She coached students who wrote mathematics competitions and supervised curling and school dances.

Katherine's phase-one class consisted of twenty-three students. The group met six times a week for sixty-seven minutes per class. The units covered by this semester-one class were sequences and series and permutations and combinations. In phase two, Katherine started out with thirty students. This semester-two class met for six sixty-seven minute classes per week. The units covered by this group were polynomial functions, trigonometry, and statistics.

View of writing. Throughout most of the study, Katherine viewed the aim of writing in mathematics as teaching students to produce better written solutions. She found writing activities to be incongruent with mathematics.

> In math, you don't see kids write something like that, which is really, ... more an English, Language Arts type of thing. (Katherine, Feb. 12, 3)

Katherine saw writing as transactional, intended to convey information to a person other than the author. She specifically told students to write to an audience.

> Pretend you are explaining this to someone who hasn't taken math in 20 years. (Katherine, Dec. 4, 4)

Writing was not seen as an activity with unique characteristics. Katherine wondered whether classroom discussion might accomplish the same exchange of ideas and feedback to students more efficiently.

> Talking you can follow up - tease things out. In writing you can't do that. So in writing you hope you'd verbalize it a bit more. I think it's faster and easier to have kids talk about math, because you can get a bunch of inputs at the same time - get the whole class involved in discussion. (Katherine, April 14)

Katherine's perspective was closest to a 'learning to write' view. She saw writing as a means of displaying and evaluating students' knowledge. The product, rather than the process of writing, was emphasized. These views of writing were reflected in:

1. Katherine's reasons for using writing activities in her mathematics classroom; and
2. her implementation of writing activities.

Reasons for using writing activities. In response to a notice from Alberta Education (1990) stating that language-based questions would become part of the Mathematics 30

Diploma examination in 1992, Katherine included a few questions requiring writing on tests in the 1990/91 school year.

> There was something came out from Alberta Ed about a year before ... that said ... types of questions like this could be on (the Diploma examination) and so the odd time, I would throw one on (a test) that said, "Explain in your words what something is." But very rarely. (Katherine, April 14)

Participating in this study enabled Katherine to become more experienced using writing activities in her classroom. It also provided an opportunity for her to assist students in mastering language based-questions.

> If nothing else, students will be more relaxed with the Diploma exam. (Katherine, Nov. 29, 3)

Consequently, she did not view writing as an appropriate activity for non-academic students who would not write the Diploma examination. Writing was a suitable activity for academic students. Katherine speculated about using writing activities with a class of enrichment students.

> I'd like to see my (good) classes (do some writing), especially one. Some of the kids are just so ingenious. I think it would be really interesting to see what some of their thoughts are. ... I'm not saying that Math 23's aren't thinking also. But I guess the math-related thought that would come from the (academic) students as opposed to the math-related thought from the 23's. ... I get a really big kick out of seeing things like this. (Katherine, April 14)

Katherine thought that writing activities should be included in grade 10 and 11 courses. This would properly prepare students to write responses when they got to grade

twelve. As students practiced writing, they would write more and become less intimidated by feedback on the style and content of their responses.

Implementation of writing activities. Katherine preferred using content-oriented writing activities in both phases of the study.

Table 1

Types of Writing Activities Used by Katherine

| | Phase 1 | | Phase 2 | |
|---|---|---|---|---|
| type | number | percent | number | percent |
| Content | 7 | 78 | 3 | 43 |
| Affective | 0 | 0 | 2 | 29 |
| Instructional | 2 | 22 | 1 | 14 |
| Creative | 0 | 0 | 1 | 14 |
| Total | 9 | 100 | 7 | 100 |
| number of periods | 24 | | 48 | |

Content writing prompts gave Katherine a way to assess students' knowledge and to review mathematical concepts presented in class. Katherine particularly liked using prompts that asked students to summarize a lesson or that emphasized connections between different concepts.

Instructional, affective and creative writing activities were used less frequently. While Katherine found these interesting to read and potentially useful in getting to know students, she had reservations about using them too often.

> I don't think that's going to give you any insight at all, except they hate math and their surroundings. ... Some are just belly-aching. (Katherine, Nov. 29, 1/2)

Nor did she feel non-content questions benefited students.

> I don't think it helped the students at all. ... (It's) just a fun or different way to spend 10 minutes. (Katherine, Feb. 5, 3)

Katherine was not sure that she would use non-content writing activities in future courses.

Katherine gave most writing activities at the end of the class after the lesson. She found that students often were distracted by announcements and eager to pack up their books. Some forgot to turn writing assignments in as they left the classroom.

Writing did not become an integral part of Katherine's teaching. Activities were extras in a class, rather than important parts of the lesson. Katherine did not develop strategies for using prompts that might have evolved into a planned program of writing activities. Writing was less important than other activities in her classes.

> Lately writing has not fit in well because I feel really pressed to cover the material and 5 to 10 minutes out of a class seems hard to part with. ... It just seems like I should be giving them that time to get through a few more questions in case they're going to ask something. ... It feels like it's time consuming. (Katherine, April 14)

While most of the writing prompts used were supplied by the researcher, Katherine did make a few prompts of her own. The number of writing activities used decreased as the study

progressed. Katherine's initial goal of three writing activities per week quickly became two per week. In phase two, this declined to one a week. Eventually a month passed in which no writing activities were used.

This was partly due to a crisis in Katherine's family in early March. When Katherine returned to teaching, she was naturally upset and, as with any prolonged absence, behind schedule. Later, Katherine did include three more writing activities.

Katherine felt that it was necessary for student responses to be marked, even if given in class. She explored alternate ways to give feedback to students: making written comments to individuals; copying excellent solutions for students to see and discuss; and posting solutions on the bulletin board. However, she kept coming back to grading student responses.

> I wouldn't mind handing this back so (students) could see I did read these and make a specific comment. ... It (is) much more meaningful and would encourage someone to write. ... I don't think just a check (mark will do). ... It could be just running off the best (response) and pointing out, "This is what it would be nice to have." ... They need feedback. ... "This probably would have been a three based on the marking system." ... I think that's what they need. Otherwise they're just doing all this writing. (Katherine, Feb. 5, 1/2)

Katherine felt most comfortable putting questions involving writing on tests. She planned to continue doing so

in the future. She was not sure she would have the time to include writing activities in class work.

Katherine concluded that content writing activities did help students to learn mathematics.

> I think that writing about concepts helps students organize concepts and forces them to present concepts clearly. (Katherine, questionnaire, 2)

Writing activities helped her to assess students' knowledge of mathematics and enabled students to evaluate their own understanding.

> Well I can see that because they've had to do it and transfer it to, hopefully, their own words and almost in a way turn around and explain it to someone else, then they do know whether they know it or not. (Katherine, April 14)

Changes in perception. Katherine's view of writing changed during the study. This may indicate that she reflected on her philosophy of mathematics and learning. Near the end of the study, Katherine developed some reservations about grading students' responses.

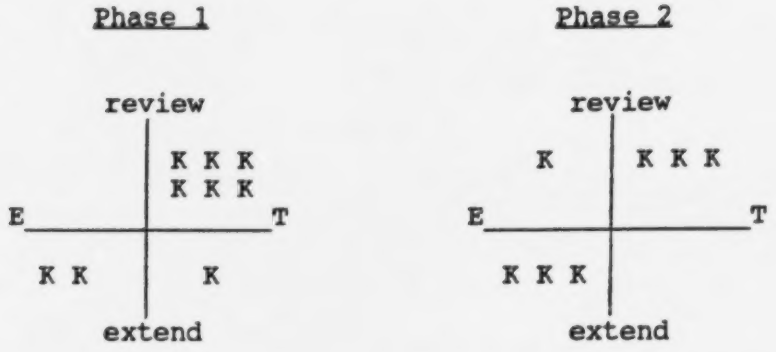
> I don't mind reading. I don't mind reading and even commenting. It's marking ... I don't think it's great to mark them. Also, it seems like the first ones were really general like, "Why are you taking Math 30?" You can't mark something like that, until they feel a bit better about what it is they're doing. But ... (students) want to know, "On a scale of ... 1 to 5, how did this compare? How (good) is this?" ... They expect to get things marked and they like to get things marked. At the same time they may be more cautious of writing ... I don't think they would be as risky if they knew it was going to be marked. (Katherine, April 14)

Katherine considered writing to be a form of evaluation, but a different type than usually found in mathematics courses. Questions requiring writing helped her to assess students' understanding of mathematics concepts, rather than their ability to perform mathematical tasks by rote memory.

> (Writing gives you) more insight into what's going on in their heads, ... (into) what they're thinking. ... (It shows) whether there's ... understanding or whether it's ... just rote memory or ... recall. ... I think it's more appropriate (as a kind of evaluation). ... I'm not so concerned with whether (students) get that (answer) right at the end. I want to know if (they) understand what should be going on to get it and why we're trying to find it. (Katherine, Feb. 26, 4)

With this shift in emphasis, the types of writing prompts used by Katherine changed. To show this, two dimensions of each writing activity were identified. First, the degree to which the writing activity was expressive (E) or transactional (T) in nature was described. Second, the degree to which the writing activity was used to review previously known concepts or to extend students' present knowledge and develop new understandings was described. These dimensions allowed each writing activity (K) used by Katherine to be placed on the grid shown in Figure 1.

Figure 1. Changes in writing activities used by Katherine




K - writing activity used by Katherine
E - expressive writing T - transactional writing
review - reviews previously learned concepts
extend - entends existing knowledge

A shift occurred from transactional writing to expressive writing. More activities were used to extend students' understanding than to review previously studied concepts. A wider variety of writing activities were utilized.

Katherine also reflected on the way expressing ideas in one's own words can cause learning. She also related the process of writing to her own experience as a learner.

> I think if they have to write about it or talk about it to someone it's really beneficial. I remember going to university and I couldn't figure something out. My Mom would just sit and listen to me explain what I was trying to figure out. By having to explain it, I'd solve the problem. And yet she had no idea what I was talking about. (Katherine, Dec. 18)

I have to think here with a pencil in my hand.
(Katherine, April 14)

Katherine's comments indicated a change in her focus when reading and discussing students' responses.

Table 2

Types of Comments Made by Katherine

| | Phase 1 | | Phase 2 | |
|---|---|---|---|---|
| type | number | percent | number | percent |
| Understanding | 33 | 30 | 19 | 15 |
| Attitude | 15 | 14 | 26 | 21 |
| Teaching | 11 | 10 | 10 | 8 |
| Reflective | 52 | 47 | 68 | 55 |
| Total | 111 | 101 | 123 | 99 |

Analysis of comments using categories defined in Chapter III indicates a significant decrease in the number of comments related to students' understanding. Corresponding increases in the number of comments regarding students' attitudes and reflection were noted. These trends may indicate a shift in Katherine's thinking. She appeared to focus less on using writing to evaluate mathematical content and more on obtaining other types of feedback.

After the study was over, Katherine continued to be interested in writing. Pointing out an article by Miller (1992), she contemplated using writing activities at the beginning of the class rather than at the end.

Katherine found that she had not only become more comfortable using writing activities in mathematics, but that

it changed her way of looking at mathematical questioning.

> Yes, it is (a real change for me). I think that the more (writing is) used the more comfortable a person is with using it. ... Before (if) I ever asked a kid to do anything, (it was like) "You must answer in a complete sentence when you're answering a word problem." ... And that was sort of the extent of it. ... I think (using writing has) been a real change for me. ... Often, I know, when I'm making up long answer questions, I'll think that, "Well, I could word this differently and then it could be a writing question." ... Instead of saying, "Identify the amplitude and period" ... (I might say) "Explain how you would identify these." (Katherine, April 14)

Katherine started to view writing as an appropriate activity for mathematics instruction, not just a type of question to include on tests.

> Yes, any alternate or different technique (like writing activities) can only enhance instruction, not alone, but in combination with existing techniques. (Katherine, questionnaire, 2)

She moved closer to a 'writing to learn' perspective, seeing that writing could assist students in their learning of mathematics.

Ian

Background. Ian had taught mathematics for nine years, five of which were substitute teaching. Most of these were at the high school level. He had been at his present school for three years. This was Ian's first time teaching Mathematics 30.

Ian had a bachelor's degree majoring in Physical

Education with a Mathematics minor. He was relaxed and friendly with his classes, talking more than giving notes. His classroom walls were covered with cartoons and mathematics posters. The room is full of plants. Ian coached badminton and was a co-sponsor of the school basketball tournament.

Ian taught two full-year classes of Mathematics 30. These classes met for three sixty-seven minute periods a week. Only one class of twenty-four students participated in phase one. The units covered by this group were trigonometry and exponents and logarithms. The other class contained nineteen students. Both classes participated in phase two of the study. Units covered were statistics, sequences and series.

View of writing. Ian consistently viewed writing as a way to get students to think about the mathematics they studied in class. Students wrote for themselves rather than for the teacher. The primary type of writing was expressive, rather than transactional.

Ian's perspective on writing was consistent with a 'writing to learn' view. He emphasized the process, rather than the product of writing. There was no one correct answer in students' responses. What writing could do for students' understanding was more important.

> It depends if you want good writing or if you want them, ... to understand what's going on in mathematics. And I would rather use it as a tool to get the understanding than, well, "I want a perfect essay on the quadratic formula." So, I'm looking at it more as a tool than as an actual end result. ... The end result is not important at all. It's getting them to try. ... But I still believe that it's the process of organizing ... that's the important part of the writing. I don't think getting a right answer is the important thing. (Ian, Apr. 22)

Ian recognized that writing was a valuable means of clarifying one's thoughts.

> It's a really good thing! ... When I have to teach something, I have to write stuff down myself once. Otherwise I really don't know it. ... It's the same thing (for students). The more times you can get (students) to write (about a concept) and think about it, the better off they are. ... Writing down stuff helps you internalize it. (Ian, March 2)

Through thinking and organizing one's thoughts, students would learn mathematics better.

> I don't think (the students) appreciate they're actually going to learn something from (writing), in terms of organizing their thoughts. ... They'll actually sit back and say to themselves, "Gee, I didn't know the answer to this question, but maybe I'll write it out now and see if I can (get it)." So if you can catch (the students) ... into actually writing down and organizing things, I think they're going to learn. (Ian, Dec. 18)

These views of writing were reflected in :

1. Ian's reasons for using writing activities in his mathematics classroom; and
2. his implementation of writing activities.

Reasons for using writing activities. Ian used writing activities with his Mathematics 30 classes because he wanted to reinforce mathematical concepts.

> I'm forcing them to think about it and the reason I'm forcing them to think about it is because, "Gee, we're going to use that idea later on and I want you to remember that." ... If it was an important concept such that I wanted to make sure everyone really understands it, then I'd get them to write it. ... If it's written, it will be reinforced a lot better by a wider mass of people, a larger population. (Ian, April 22)

Ian also wanted to find out about students' feelings and attitudes. He anticipated getting to know students in a more personal way.

> (It will be) interesting to see the responses next semester. It's a good class, but quiet. I'll probably appreciate them more if I see some writing. I don't get any interaction at all. They're just out there. I don't know them personally at all. They're quiet in nature. I'm interested to ... meet them through this (writing). Some of them ... I haven't talked to yet. (Ian, Nov. 27, 2)

Writing activities were also used to broaden students' conceptions of mathematics. Ian felt that students gave too much importance to using the right formula and getting the right answer. He hoped that writing activities could serve as a way to show students that mathematics involved more.

> (Writing is) not a right/wrong thing. So (the students) don't get this impression of "What is the right answer?" (Ian, April 22)

Implementation. Ian used content writing prompts more than any other type.

Table 3

Types of Writing Activities Used by Ian

| | Phase 1 | | Phase 2 | |
|---|---|---|---|---|
| type | number | percent | number | percent |
| Content | 5 | 71 | 5 | 83 |
| Affective | 1 | 14 | 1 | 17 |
| Instructional | 1 | 14 | 0 | 0 |
| Creative | 0 | 0 | 0 | 0 |
| Total | 7 | 99 | 6 | 100 |
| number of periods | 12 | | 24 | |

Content prompts were used to reinforce topics presented in class and to give Ian feedback on students' understanding of concepts. He liked to make up impromptu writing activities in the middle of a class to check on students' understanding. Topics were retaught immediately if necessary. Other activities were administered one or two days later to see whether students had internalized the concepts involved and to reinforce their learning. Most writing activities were administered between the lesson and the time during which students worked on assignments. Writing activities took about ten minutes each.

Ian liked the fact that questions requiring writing had a different emphasis than the usual question asked in mathematics exercises.

> I wanted these guys to think (about) a little bit (more than) just doing questions and this is different. Talking about what the graph (of a sequence) looks like is different than "What's the fifth term? What's the seventh term?" (The unit on) sequences ... tends to get in that mode. (Ian, March 18, 9)

Ian also felt that affective writing activities might assist students to develop good work habits.

> Writing is another way to let them know that they have to do more work, that they're responsible, that I can't do it all. (Ian, Nov. 27, 2)

But he found that writing activities emphasized students' lack of effort and responsibility even more than regular classroom interaction.

> Anytime I read things from (the students), it emphasizes how much they bother me. ... 90% of them say they should have done more work, but they still didn't do their homework last night. I ask myself, "Why am I there?" ... The experience of writing is not negative, just ... the reinforcement of the attitude that I see. (Before, I) only (saw it) three times a week, now I see it five times a week. ... When you read something, it ... hits home more than in class. When you're teaching, you can ... ignore it. (Ian, Dec. 18)

In spite of this, Ian found affective questions to be of value. While it was important to know whether students understood a topic, it was also valuable to know how they felt about the course. Because students could not be evaluated, they found affective writing prompts less stressful to respond to, particularly at the beginning of the study.

Ian found that writing activities fit very naturally with his usual style of teaching. His casual integration of writing activities into instruction indicated a high degree of comfort and acceptance of the technique. Indeed, Ian indicated that while he had not previously used writing

itself, the types of questions asked were not a great departure from those previously used. Nor did writing activities slow his teaching down.

> The time they would normally be working on questions is the time I do the writing. So it has no effect on me. I just think it's part of the course. A lot of times in a class, ... I'll ask a question which is the same as ... a writing question. ... So, no, it doesn't affect the way I do things at all. (Ian, Feb. 14)

Part way through phase two, Ian discovered that he had, in fact, been having students write to him for several years.

> I never thought of this (before) as a writing assignment, but I have a big binder (in which students) write their excuses for why they're late or away everyday. (Ian, March 2)

Ian found that writing activities had several major advantages over other types of teaching techniques. First, writing made students accountable for their learning. All students were forced to participate.

> If you get them to write down something, you're invading their little bit of privacy, but you're forcing them to (work). Reality strikes. ... There's no escaping the 'wrath of writing'. (Ian, March 2)

Second, he felt that writing reinforced understanding more than other modes of communication.

> I think that learning takes place by reading and listening, but that understanding takes place in the process of writing. (Ian, questionnaire, 2)

Third, Ian liked the fact that he could sit down and look at student responses later.

These may be the reasons why Ian chose to use writing activities with both of his Mathematics 30 classes. Originally, one class was scheduled for phase one and the other for phase two. Ian liked writing activities well enough to continue with the phase-one group. Ian used writing activities with other classes during the study and planned to use writing prompts in future courses.

Ian did not like marking students' written responses.

> I prefer not to mark these. ... It ... stifles (the students). ... I like to use writing as a learning thing, not a test, not marked. (Ian, Dec. 4, 3)

He did feel, however, that it was important to acknowledge student writing.

> I would make sure I read them all. If it's content, I would make comments about what I'm looking for in an answer. I would make sure I ran a couple where I actually marked them. (Ian, April 22)

Changes in perception. Ian's ways of using writing activities became increasingly complex over the two phases. Prompts were designed to meet specific instructional needs. Ian found that writing strengthened his philosophy of mathematics teaching.

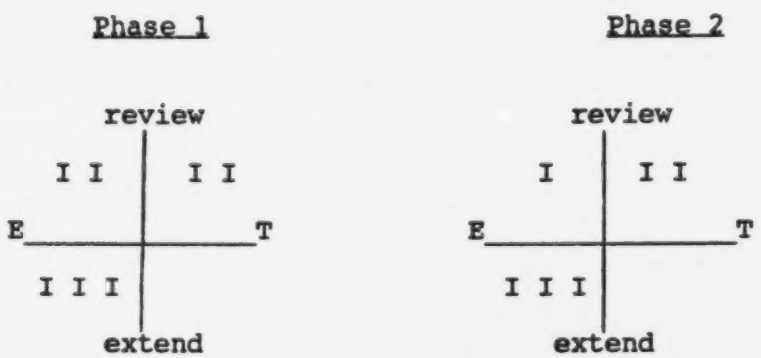
> It has reinforced my belief that understanding is the key and writing emphasizes understanding. (Ian, questionnaire, 2)

Rather than changing his emphasis, writing gave him another tool to use in teaching.

> It changed a lot in the sense that I wouldn't give a writing assignment per se. But I've always tried to ask thinking questions, just not in written form. So in terms of the actual writing of stuff, it's changed everything. In terms of how I approach things, it hasn't changed anything really. (Ian, April 22)

Types of writing activities used by Ian did not change significantly over the study. Two dimensions of each writing activity were identified. The degree to which the writing activity was expressive (E) or transactional (T) in nature, and the degree to which the writing activity was used to review previously known concepts or to extend students' present knowledge and develop new understandings were described. Each writing activity (I) used by Ian to be placed on the grid shown in Figure 2.

Figure 2. Changes in writing activities used by Ian




I - writing activity used by Ian
E - expressive writing T - transactional writing
review - reviews previously learned concepts
extend - entends existing knowledge

In contrast, the information Ian perceived in them and commented on did change.

Table 4

Types of Comments Made by Ian

| | Phase 1 | | Phase 2 | |
|---|---|---|---|---|
| type | number | percent | number | percent |
| Understanding | 16 | 12 | 38 | 22 |
| Attitude | 55 | 42 | 24 | 14 |
| Teaching | 17 | 13 | 16 | 9 |
| Reflection | 32 | 32 | 96 | 55 |
| Total | 130 | 99 | 174 | 100 |

There was a decrease in the number of comments dealing with students' largely negative attitudes. In the second half of the study, Ian noted an improvement in students' attitudes. It is not clear whether this was due to attrition, maturation, or the effects of writing. An increase in the number of reflective comments was also noted. This was consistent with Ian's increased use of strategies for writing in the second phase.

Ian felt that writing activities were a powerful addition to his regular teaching style. He found it of benefit to his students and to himself.

> (It benefits students) because they actually sit down for a few minutes and think... A lot of times they don't sit and think about what they're doing. They just do it. ... If they have to write it ..., then they actually have to think about what they're going to write and they write it. ... (It's) forcing them to take 10 minutes and actually think about something. I really don't think that in math they do that that often. (Ian, Feb. 14, 5/6)

Ian's reasons for using writing activities evolved over the course of the study, becoming increasingly complex and fully integrated into instruction. By administering prompts to classes not participating in the study, he demonstrated his appreciation of the strengths of the technique. He appears to have reflected on the use of writing in mathematics classes and on his conceptions of mathematics and mathematics teaching.

## Mary

Background. Mary had taught for 30 years in both junior and senior high schools. In junior high, she taught math and a variety of other subjects. The last 18 years had been spent teaching mathematics at the high school level. She had been at her present school for two years.

Mary had both a bachelor's degree in science and a graduate degree in education. She recently had a sabbatical from teaching and in 1990/91 took part in another research study. She had high standards of performance and behaviour for her students. Her classroom was attractively decorated with posters and well equipped with books and supplies.

Mary's phase-one class had nineteen students. This semester-one class met six times a week for sixty-seven minutes a period. The class worked through units on statistics, sequences and series, and permutations and

combinations during phase one. Mary's phase-two class contained twenty-seven students. This semester-two class again met for six sixty-seven minute periods a week. The group studied polynomial functions and trigonometry during phase two.

View of writing. Mary saw writing as a way to help students reflect, not only on the content of the course, but on the process of learning mathematics as well. She hoped that through writing, students would consolidate their understanding of mathematical concepts and acquire tools for learning.

Students responded with expressive rather than transactional writing. They wrote for themselves, rather than for an audience.

> You write it for your own personal need. (Mary, March 19, 10)

Answers were not expected to be polished, but, rather, to be personal attempts at searching for understanding.

> I said we would come back to it. It wasn't a final answer, but a starting answer. That's a start. (Mary, Feb. 11, 3)

> "Don't worry about a right or wrong answer. Just write anything you can think of." So that's what I said to them. "Just tell whatever you want to say. There's no right or wrong answer." (Mary, Nov. 26, 2)

Mary viewed writing as 'writing to learn'. She thought that what the process of writing could do for students was more important than the product of writing. Through writing,

students could learn to use mathematical language fluently, think about concepts for themselves, and construct their own understanding of mathematics.

> I find that to express yourself you have to be able to analyze and see what's really happening. If I could look at something and put it in my own words I knew what I was talking about. (Mary, Dec. 3, 4)

> To me you have to be able to verbalize . If you understand something, you can verbalize it. That's why you can teach it probably. It causes you to see a flow in your thought or whether you have a flow in your thoughts. It forces you to focus on your vocabulary. Can you communicate with the peculiar language that (mathematics) has? (It) forces you to break things down, put them back together again. Sometimes it slants (in) a different way what you're being asked to do. It's not like the questions in the textbook. (Mary, Feb. 25)

Mary's view of writing also contained elements of the 'writing to think' perspective. She hoped to empower students as learners by giving them analytic tools. She wanted them to think for themselves and take more control of their learning.

> I'm in a quandary as to how to get students to review instead of me doing all the work. My attempt was for them to put it together, instead of me. That was important for me as a learner. (Mary, Dec. 9, 6)

It was also important for students to examine themselves and to reflect on what they were doing in mathematics.

> I guess it's OK to ask a means-to-an-end question, but it would be kind of nice if they reflected a little bit and thought about themselves, rather than just what they hope to get out of the math. (Mary, Nov. 26, 1)

These views of writing were reflected in :

1. Mary's reasons for using writing activities in her mathematics classroom; and
2. her implementation of writing activities.

Reasons for using writing activities. Mary had a wide variety of reasons for using writing activities. Her first writing question was administered a month before the start of the study. It had been used to gain information on students' thoughts and feelings about mathematics.

> I did that ... a couple of weeks ago. ... I said, "When someone says the word math, what do you automatically think of?" ... "I'm weak. I don't like it. I've never been good at it." That kind of stuff. You could see where they were coming from. (Mary, Nov. 26)

Mary also used writing activities because she wanted students to construct their own understanding of mathematical concepts.

> I think that it really forces someone to rethink what they are doing and to use the vocabulary. ... It shows whether someone knows what's going on. If you know that you have to verbalize it, I think it would force you to think through what's happening. (Mary, Dec. 17)

Some writing prompts were used in order to enable students to manage their own learning. Mary hoped to teach them metacognitive strategies and thinking skills.

> I found that ... when I had difficulty learning something, I had to take apart the stuff and put it together and literally talk to myself. ... "I did this and then this. And I can see this, this, and this." And I found that it was a tremendous tool for me. ... I know that (writing) helped me. I'm hoping it can transfer to a few of them. (Mary, April 14)

Implementation. In both phases of the study, Mary used content writing activities more that other types.

Table 5

Types of Writing Activities Used by Mary

| | Phase 1 | | Phase 2 | |
|---|---|---|---|---|
| type | number | percent | number | percent |
| Content | 9 | 90 | 8 | 62 |
| Affective | 1 | 10 | 2 | 15 |
| Instructional | 0 | 0 | 2 | 15 |
| Creative | 0 | 0 | 1 | 8 |
| Total | 10 | 100 | 13 | 100 |
| number of periods | 24 | | 48 | |

Content activities gave students the opportunity to rethink mathematical concepts presented in class. Mary used prompts that invited students to compare concepts and connect ideas together.

> (There are) some stages where it's really nice to do (writing) at that point. When you realize that you've come to a ... good summarizing point, where ideas come together, or where there's something in particular that you've done and you want to make sure that it's (emphasized). (Mary, Feb. 25)

Mary found that content writing activities could successfully replace or supplement some of her usual instructional techniques.

> I'm getting to feel that the ability to write this out is even better than some of the applications. (With) an application, (you get a) respect for where you use (a concept). ... But, if you can express yourself and think of all the parts that lead in together, I think you really know what you're doing. (With an) application, (you do) not, really. By the time you wade through all

> the different (types of questions), you can lose sense (of what is going on). (Mary, Dec. 9, 5) (emphasis added)
>
> I feel that it (writing) was valuable for pulling ideas together, rather than (doing) more drill. This might be a better exercise at lots of points throughout (the course). (Mary, Dec. 9, 6) (emphasis added)
>
> I like this (writing). I've been doing homework checks. I might switch to these kinds of reactions for a while. ... It forces students to verbalize. It forces them to focus more on what they're doing. (Mary, Dec. 17) (emphasis added)

Mary thought that one of the main advantages of using questions requiring writing was that students were individually accountable for the work. They couldn't sit and "escape" as they often could in other classroom activities.

Mary experimented using writing activities at the beginning of a class to review the previous day's material and set students up for the lesson to follow. She also used writing at the end of a class or a unit to summarize the material covered. Other writing activities asked students to generalize mathematical concepts. Mary wanted students to get an intuitive feeling for concepts from other questions involving writing.

Mary also used content questions in an attempt to develop students' metacognitive skills. Questions asking students to identify strategies for learning or for preparing for tests were most common. Mary thought that students were often lacking in this area.

> They don't have strategies. ... When they say "strategies", it's always ... "What would you do to improve your mark? Study harder, do more questions." ... I said, "Well, that's a general thing. We want to get something specific." ... I've been trying to show them that ... when you're doing something, don't just do the mechanics. You have to look at what happens. (Mary, March 19, 11)

> I used to study for an exam and say, "Well, the teacher did this kind and this kind and that kind." So in my notes I'll single those out. And, "I had trouble with this one" and single that out. (Mary, Feb. 4)

In phase 2, Mary also decided to use content-oriented writing activities as part of student journals.

> I've been thinking of having them write a journal everyday. "What did you do yesterday?" Or once a week. I'm beginning to wonder if that's not a good idea. Maybe three times a week. They could summarize topics we covered. "What do you think you know and what do you think you don't know? What's still weak?" I feel that that could give you some very valuable feedback or for them even crystallize where they've been. I don't think that some kids know where they've been. (Mary, Feb. 4)

Mary initiated these journals with her Mathematics 30 class, but was soon using them with all her other mathematics classes. Students summarized material covered in a class at the end of the period or at the start of the next day. One or two week's work could be summarized concisely on one page.

> I'm trying to get a set of notes for them. Whether it will serve or not I don't know, but at least they'll know the major headings. (Mary, Feb. 11)

While, initially, journals were collected every day, Mary found every two days more practical. Mary found that journal writing on a regular basis caused a change in students' attitudes.

> I'll tell you what's happening. ... The first couple of times it was sort of, "Oh, this. Oh here it is again." And they'd scurry into class and start flipping through pages to see what it is so they'd be ready for the next one. (The students) said to me, "This is pretty hard to do". ... I'm not sure if it's the wisest thing, but it got them scurrying. ... It was happening. ... It was. You can see all the papers opening up and (hear) whisper, whisper. "Do you think she'll ask this? Do you think she'll ask that? I laughed when I saw that and I thought, "It does work. There's something there." (Mary, March 2)

Moreover, several students mentioned using their journals to study for tests.

Mary graded some of the journal entries and commented on others. But she found that it was hard to find the time required to do this. Even managing the volume of paper was difficult.

In spite of these problems, Mary felt that journals were worthwhile. They were a way of communicating with students who wouldn't say anything in class and of providing individual remediation.

> (The students) are so weak. ... They don't talk. And finally ... I said, "OK, each day you have to summarize the highlight of the class and you have to tell me something that you did not know". That's the only way I can get feedback. And it's quite good. I can write back (to the students). ... It's in the last five minutes and these kids will do it. So that's what I'm doing now. (Mary, April 14)

Non-content writing prompts were also used. Some gave information about students' personalities, attitudes and feelings. Mary found this type of feedback particularly useful at the beginning of the semester, when she looked at

students' expectations in conjunction with their marks from prerequisite courses.

> If you don't ask (about their feelings), they're just bodies there and you're just delivering information and trying to give help. ... I think it gets you at the first part (of the course) to identify something with that name, with that face, a bit sooner. (Mary, Feb. 4, 1)

Mary used instructional and affective prompts to get feedback from students about which topics and which instructional activities they found most interesting.

> Since it was a different kind of class, I wanted to see what their reaction was, ... what they liked from today's class. So they all concentrated on the activity (with graphing calculators), but when (the question) said, "What did you not like?", they all concentrated on the test. (laughs) ... Certainly there was something from the activity that (the students) didn't like either. I would have liked feedback on that. (Mary, March 2, 9)

Several affective and creative writing prompts were used to get students to focus on how they approached learning mathematics.

> I think you could wake some of them up before it's too late. (Mary, Feb. 4)

Students found it easier to respond in writing because it was more private than speaking out in class.

> You don't get personal reactions from kids as a rule at all and that's one thing you can get at with these (writing activities). ... There are some surprises (from) kids who don't respond in class. You can pick it up on paper. ... They might say it on paper, but they won't say it out loud. (Mary, Dec. 17)

Other creative and affective activities were given to help students cope with their frustration over the difficulty and complexity of some of the mathematics studied.

> They were getting frustrated. I thought it was time to (do something about it). And the next day they walked in. Nobody said anything too much. We just went on our way and we did it. (Mary, March 19, 11)

Mary found that she could comfortably do two writing activities per week, unless a holiday or special event intervened. At this rate, writing activities did not slow instruction down. Each writing activity took about ten minutes. Writing activities got a good response from the students and improved their writing ability.

Mary felt that student responses should be commented on, but not marked. She did not see evaluation as an important function of writing activities. Mary decided that her time was spent better making comments to students.

> (Writing is) a different way of giving feedback (to students), especially if the response is not marked like an exam question. (It is) not good, if ... you know (a student's problem), but do nothing about it. ... (It's) the same response then as an exam question. (Mary, questionnaire, 1)

Mary made comments regularly on students' responses. It took her about ten to fifteen minutes per writing activity. She made what she called "kind reminders" to get students to feel more comfortable with writing and more proficient at doing it.

> Basically, in a class, I think that's the best thing to do at the beginning when learning. Maybe let them write the first one and say, "Now here are some guidelines, so you can learn each time you write." (Mary, Dec. 9, 5)

Mary found that comments provided personal, one-to-one contact with students. Such contact was not always possible in class on a daily basis. She felt it was essential that the students know that the teacher read their responses.

Mary put only one writing prompt on a test during the study. She did try marking a few writing activities used in class, but she had reservations about doing so.

> I find these hard to mark. You need to be very careful about the wording so that you can narrow down what (the answer) is. The key you need to define carefully and stick to it, to mark (the question) according to the key. So the wording is important. Every time I look at these I change my mind quite easily by one mark. (Mary, Dec. 3, 4)

Mary was also concerned about the time needed to regularly mark writing activities on top of the other demands of teaching.

> These are all worthwhile things to do. I'm just looking at, "Can I mark that (writing)? Can I mark problem-solving? Can I mark homework checks?" ... You see, I'm pressured for time. ... The thing that bothers me now is ... the pressure of time. ... So I really find that I'm not happy with this year at all. I'm finding that ... more and more all I do is go home and then live in a cocoon. (Mary, Nov. 26, 2)

Lack of time produced real frustration for Mary, as did students' attitudes to learning. Many students were not able to see the relationship between the work they did in mathematics class and the marks they got. Mary repeatedly

mentioned her frustration at seeing students promise to work harder and never following through.

> "I should have studied more." To me they say it over and over and over again, but they don't do anything about it. (Mary, Feb. 25, 6)

In spite of these problems, Mary felt that writing activities provided a good addition to her teaching repertoire.

> I just think the biggest thing is ( that writing) gives the (students) a chance to see whether they can consolidate some things or not. And what they do for themselves is going to be (better). In a desk, if you don't say anything, they think they're hidden from you. ... But this way, they know on a daily basis (that you're) pretty close, (which is hard to do) especially when you see 120, 150 (students). (Mary, March 2)

Mary found that writing activities fit in naturally with her usual teaching style. She intended to incorporate writing in future courses, but was not sure that she would use it all the time.

> Chances are (that in) April, I'm going to switch to some problem solving techniques. ... If you run out of time, you're going to do things in isolated packages. ... You can't spread yourself that thin. I can see myself almost doing clusters. Two months on cooperative learning, two months doing homework checks, two months writing journals, and two months on problem solving. You can't do (them) all at the same time. (Mary, Feb. 11)

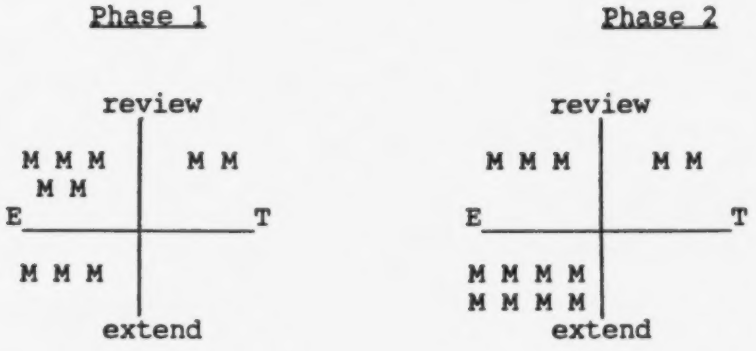
Mary used writing activities not just to meet the demands of the study, but to meet what she saw as the demands of her teaching. She used writing creatively, inventing many of her own prompts and initiating the use of journals.

Sophisticated strategies were developed for incorporating writing activities into lessons.

Changes in perception. The ways in which writing activities were used changed over the course of the study. Mary moved from using writing activities as individual activities, disjoint from her instruction, to using a program of writing fully integrated into her teaching. She experimented more with different uses and types of writing activities. Her innovative use of student journals far exceeded what was required for the study. Mary really took ownership of the program of writing activities, using them more for her own purposes than for those of the study.

Analysis of writing activities illustrates these changes. Two dimensions of each writing activity were identified. First, the degree to which the writing activity was expressive (E) or transactional (T) in nature was described. Second, the degree to which the writing activity was used to review previously known concepts or to extend students' present knowledge and develop new understandings was described. These dimensions allowed each writing activity (M) used by Mary to be placed on the grid shown in Figure 3.

Figure 3. Changes in writing activities used by Mary




M - writing activity used by Mary
E - expressive writing T - transactional writing
review - reviews previously learned concepts
extend - entends existing knowledge

There was a shift away from writing activities that reviewed material to writing activities that extended and developed new understandings.

Mary showed that she had taken ownership of writing activities by seeking out new sources of information on writing in content areas. She requested joint interviews with other participants, in order to find out what writing they were doing with their classes. She went to a session on journal writing during the local teachers' convention. Ideas from this talk were incorporated in student journals and shared with other participants.

Mary also began to find opportunities for getting students to write where she had not seen them before. She found that several textbooks which had been in use for several years contained questions that could be adapted as writing prompts.

In phase two, Mary used writing activities, not just with her Mathematics 30 students, but with all of her classes. She continued to use writing activities after the study was over.

The kind of information Mary perceived in student responses changed throughout the study.

Table 6

Types of Comments Made by Mary

| | Phase 1 | | Phase 2 | |
|---|---|---|---|---|
| type | number | percent | number | percent |
| Understanding | 67 | 39 | 49 | 21 |
| Attitude | 20 | 12 | 29 | 12 |
| Teaching | 32 | 19 | 21 | 9 |
| Reflection | 52 | 30 | 139 | 58 |
| Total | 171 | 100 | 238 | 100 |

There was a shift away from comments dealing with students' understanding of concepts. A decrease in the proportion of comments dealing with teaching was also noted. Mary's comments and perceptions changed their focus as she reflected on mathematics, teaching, and writing in mathematics.

Mary herself thought that changes had occurred in her questioning and in what she required of students.

> You start thinking of different ways of focusing on a question, different ways of questioning ... you don't find in books. ... Kids have to focus, evaluate what they're doing, what they might do, why, consider alternate methods, special cases. Kids think in a slightly different (way). (Mary, Feb. 25)

> It causes you to change your focus once in a while. It gives you variety at the same time it gives (students) variety. It's like another tool in your repertoire, I guess. (April 14)

Mary not only mastered the uses of writing activities suggested by the researcher, but extended these in creative and innovative ways. She personally related the writing process to her own experiences as a learner. There is much evidence showing that she did reflect critically on her teaching of mathematics.

## Summary

These three participants had widely varying backgrounds and experience. They had from nine to thirty years of teaching experience. Their academic qualifications included majors in physical education, science, and mathematics. One had taught Mathematics 30 for many years; another taught Mathematics 30 for the first time during the study. Even their experience with writing varied. One teacher had used some writing activities in the previous school year. Another used writing for the first time during the study.

Participants' views of writing varied greatly. Students used both expressive writing and transactional writing. Some teachers emphasized the product of writing; others the process. Writing was used both to evaluate students and as a mode of learning. All three paradigmatic views of writing in content areas were represented. Katherine saw writing in mathematics as 'learning to write'. Ian saw writing in mathematics as 'writing to learn'. Mary saw writing both as 'writing to learn' and 'writing to think'.

A variety of reasons for using writing activities in mathematics were also uncovered. Initially, all participants agreed to become part of the study and try writing activities in response to the Alberta Education decree. If questions involving writing were to be part of the final Diploma examination, then students needed to be prepared to answer them. Teachers saw the study as a chance to get information about writing and help in incorporating it in their classrooms.

But as the study progressed a number of other reasons for using writing activities evolved. Writing was used:

1. to elicit information on students' attitudes and feelings, particularly with respect to mathematics and mathematics instruction;
2. to help students in constructing their own understanding of mathematical concepts;

3. to empower students as learners by assisting them to reflect on and develop metacognitive strategies for learning mathematics and by giving them a means of evaluating their own learning;
4. to broaden students' conceptions of mathematics;
5. to assess students' understanding of mathematical concepts; and
6. to assess the effectiveness of classroom instruction in general and different teaching techniques in particular.

In spite of the major differences among teachers, all three participants found writing a useful addition to their teaching repertoire. Teachers found that writing activities took about ten minutes each. Two could be managed per week (six classes) unless something interfered. Mary and Ian felt that, at this rate, writing did not slow their teaching down. Katherine disagreed.

All teachers used a variety of kinds of prompts.

Table 7

Writing Activities Used by Teachers during the Study

| type | Katherine | Ian | Mary |
|---|---|---|---|
| Content | 10 | 9 | 17 |
| Affective | 2 | 3 | 3 |
| Instructional | 3 | 1 | 2 |
| Creative | 1 | 0 | 1 |
| Total | 16 | 13 | 23 |
| number of periods | 72 | 36 | 72 |

Content writing activities were used most often by the three participants. Popular prompts asked students:

1. to summarize lessons or units;
2. to make connections between topics;
3. to explain concepts in their own words;
4. to compare concepts;
5. to categorize problems;
6. to outline strategies; and
7. to reflect on study skills.

Affective prompts were administered by all participants. Teachers used them to gather information on students' attitudes and feelings:

1. at the beginning of a course;
2. when students appeared to be frustrated; and
3. when new activities were introduced in class.

Teachers found student responses to affective prompts gave them more empathy towards students and allowed them to get to know students in a personal way not usual in a mathematics class.

Instructional writing activities were also used by all participating teachers. Teachers administered such prompts to get students' opinions and feelings about:

1. mathematics instruction and teachers; and
2. new teaching activities tried in the classroom.

Katherine and Mary each used one creative writing activity to get students to reflect on the nature of mathematics. Ian did not administer any prompts of this type.

All teachers used writing activities in classes other than the participating Mathematics 30 groups. All agreed that they would continue to use writing activities for the rest of the year and in future courses. They felt that incorporating writing in earlier grades would be beneficial.

Writing activities were used by all participants in class and on tests. Katherine preferred using writing prompts for evaluation. Ian and Mary preferred using writing prompts in class to help students learn. All participants experimented with marking student responses. None felt particularly comfortable doing so.

Participants strongly felt that it was necessary to give feedback to students about their responses. Comments were seen as important for helping students improve their written responses. But more importantly, comments let students know that their responses were read and taken seriously.

Teachers reflected on the use of writing activities in mathematics. They were able to identify a number of reasons for using it in preference or in addition to other instructional techniques.

1. The act of writing made all students actively participate and be accountable for their learning.
2. Writing activities gave teachers a different type of information about their students than was usually available to them.
3. Written responses were available for reading and reflection by both students and teachers.
4. Some students responded to questions involving writing when they did not respond to other activities in class.
5. Writing reinforced students' understanding more than other forms of communication did.

Several problems were found to be associated with incorporating writing activities in mathematics instruction.

1. Teachers found it depressing to read over and over again about students' intentions to work harder when this never materialized.
2. Participants were overwhelmed by the number of demands placed on them as part of their regular teaching duties. Writing was one more thing to think about and plan for in an already busy schedule.
3. While teachers believed it important to read and comment on student responses. they found that the time required to do so was sometimes difficult to find.
4. Duplicating, handing out, collecting, and storing students' written responses involved teachers in a

"paper blizzard". The number of sheets involved was sometimes overwhelming.

Several changes occurred to show that teachers may have reflected on their concepts of mathematics and mathematics teaching. There were changes in the types of writing prompts used by participants, as revealed in Table 8.

Table 8

Differences in Writing Activities Used by Teachers in the Two Phases of the Study

Phase 1

| | Teacher | | | | |
|---|---|---|---|---|---|
| type | Katherine | Ian | Mary | total | percent |
| Content | 7 | 4 | 8 | 24 | .800 |
| Affective | 0 | 1 | 1 | 2 | .067 |
| Instructional | 2 | 1 | 0 | 3 | .100 |
| Creative | 0 | 0 | 0 | 1 | .033 |
| Total | 9 | 6 | 9 | 30 | 1.000 |

Phase 2

| | Teacher | | | | |
|---|---|---|---|---|---|
| type | Katherine | Ian | Mary | total | percent |
| Content | 3 | 5 | 9 | 17 | .607 |
| Affective | 2 | 2 | 2 | 6 | .214 |
| Instructional | 1 | 0 | 2 | 3 | .107 |
| Creative | 1 | 0 | 1 | 2 | .071 |
| Total | 7 | 7 | 14 | 28 | 1.000 |

There was an overall shift away from content writing activities to other types of writing prompts, particularly affective questions. This indicates that as teachers became more experienced with the use of writing activities, they had the opportunity to reflect on the other benefits accruing

from their use. Their views of writing as an activity in mathematics instruction were modified. This may also indicate a broadening of their conceptions of mathematics as well.

Other changes in writing activities were evident from both qualitative and quantitative data. Fewer writing activities per week were administered. Phase one lasted for four weeks, while phase two continued for eight. Consequently, twice as many writing activities would be expected. Instead, the number decreased.

While teachers used fewer writing prompts, they used them more thoughtfully. Writing activities became an integral part of instruction. Teachers developed complex strategies for making full use of writing prompts. Rather than using writing activities in isolation from instruction, they began to use them intentionally as part of their program of instruction. This indicates teachers were reflecting about the relationship between writing and mathematics instruction in general.

The characteristics of writing activities used also changed throughout the study. Teachers used less transactional writing and more expressive writing. The emphasis in writing shifted from reviewing previously learned concepts to constructing new understandings.

There was a change in the information teachers perceived in student responses as indicated in Table 9.

Table 9

Types of Comments Made by Teachers

Phase 1

| | Teacher | | | | |
|---|---|---|---|---|---|
| type | Katherine | Ian | Mary | total | percent |
| Understanding | 30 | 12 | 39 | 81 | .270 |
| Attitudes | 14 | 42 | 12 | 68 | .227 |
| Teaching | 10 | 13 | 19 | 42 | .140 |
| Reflection | 47 | 32 | 30 | 109 | .363 |
| Total | 111 | 130 | 171 | 300 | 1.000 |

Phase 2

| | Teacher | | | | |
|---|---|---|---|---|---|
| type | Katherine | Ian | Mary | total | percent |
| Understanding | 15 | 22 | 21 | 58 | .193 |
| Attitudes | 22 | 14 | 12 | 48 | .160 |
| Teaching | 8 | 9 | 9 | 26 | .087 |
| Reflection | 55 | 55 | 58 | 168 | .560 |
| Total | 123 | 174 | 238 | 300 | 1.000 |

Changes in categories vary from teacher to teacher. A consistent increase is shown, however, in the number of comments related to teacher reflection. Comments made in phase two were more thoughtful, and showed more teacher reflection on writing and teaching mathematics.

Changes in writing prompts used and in teacher comments revealed changes in their views of writing in mathematics. Katherine moved from a 'learning to write' perspective closer to a 'writing to learn' perspective. Ian remained firmly

located in the 'learning to write' paradigm. Mary strengthened her commitment to 'writing to think'.

The fact that writing became an accepted teaching "tool" for participants indicates that they reflected on their views of mathematics and mathematics teaching. Evidence exists to indicate that writing activities did provide teachers with the opportunity to reflect on their conception of mathematics and mathematics teaching, and that teachers did, in fact, take advantage of this opportunity.

# CHAPTER VI - SUMMARY AND DISCUSSION

## Introduction

> We shall cease from exploration
> And the end of our exploring
> Will be to arrive where we started
> And know the place for the first time.
> (Elliot, 1936, p. 74)

This study has explored the effect of writing on teacher awareness and reflection. Chapter VI will trace these explorations. The purpose and design of the study will be briefly reviewed. A discussion of the results follows. Finally, suggestions for further research will be presented.

## Review of the Study

There has been a move in the last decade toward the inclusion of writing by students as part of mathematics instruction. The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) state that "representing, talking, listening, writing, and reading are key communication skills and should be viewed as integral parts of the mathematics curriculum" (p.27, emphasis added).

When a new instructional technique is introduced, it is necessary to determine whether it benefits students in learning and whether it benefits teachers in teaching. Most of the research investigating writing in mathematics has focused on the gains of the student. While this is of

paramount importance, it is also necessary to find out what writing offers from the teacher's viewpoint.

> We are unlikely to make fundamental changes in instruction simply by changing curricula and activities without attention to the purposes the activities serve for the teacher as well as for the student. (Langer & Applebee, 1987, p. 87)

Previous research in the area of writing in content areas indicates that one of the purposes writing may serve is to enhance communication between students and teacher. This study examined whether this increased communication enhanced mathematics teachers' awareness. Four questions were considered.

1. Did the use of writing activities provide an opportunity for teachers to become more aware of the effectiveness of classroom instruction?
2. Did the use of writing activities provide an opportunity for teachers to become more aware of students' attitudes and feelings?
3. Did the use of writing activities provide an opportunity for teachers to become more aware of students' understandings of mathematical concepts and processes?
4. Did the use of writing activities provide an opportunity for teachers to reflect on their own conceptions of mathematics and teaching?

The study was carried out in a large, urban high school. Four teachers of Mathematics 30, the final course in the academic stream of the mathematics program, participated.

The study consisted of two phases. All four participants used writing activities in their classes during the developmental phase in November and December of 1991. Three of these teachers continued in the implementation phase in the spring of 1992. In this second part of the study, teachers began to use writing activities more purposefully, integrating them into classroom instruction.

Weekly interviews were held with teachers throughout both phases. Questionnaires were completed by participants at the conclusion of each phase. Student responses and interviews with seven students were also analyzed.

Evidence was presented to show that writing activities did provide a source of information for teachers on the effectiveness of classroom instruction. Teachers became more aware of the effectiveness of mathematics instruction, in general, different teaching techniques, instruction of specific topics, classroom communication, and instruction on writing. Moreover, teachers used writing prompts to elicit information on instruction from students and modified their instruction in response.

Student responses also provided teachers with information on students' attitudes and feelings. Responses

revealed students' emotional reactions, attitudes to studying mathematics, and conceptions of mathematics. Teachers' comments showed they were aware of this feedback. On occasion, they specifically used writing activities to elicit such information. It appeared that teachers had internalized their new insights into students' attitudes and feelings.

Written responses also provided teachers with information on students' understanding of mathematical content. Student responses contained feedback on students' background knowledge, knowledge of Mathematics 30 concepts and processes, use of metacognitive strategies, use of mathematical language, and modes of understanding mathematics. Teachers showed they were aware of this information by diagnosing areas causing students difficulty, providing remediation, modifying instruction, and changing their opinions of students.

Teachers' use of writing activities changed over the course of the study. Examination of the types of changes that occurred and the reasons teachers had for making these modifications showed that teachers did reflect on their conceptions of mathematics and teaching.

Teachers' reasons for using writing activities evolved as their experience increased. From viewing writing as a means of assessing students' understanding, teachers moved to seeing writing as a mode of learning for students and as a

way to empower students as thinkers and learners in the classroom. As teachers' conceptions of writing in mathematics changed, so too did teachers' choice of prompts and their ways of using these.

Writing moved from being distinct from other instructional activities to being an integral part of mathematics teaching for participants. Teachers views shifted away from 'learning to write' toward 'writing to learn' and 'writing to think'. These shifts revealed teachers' reflections on the use of writing activities in mathematics and mathematics itself. Changing roles of students and teachers revealed teachers' reflections on teaching.

## Discussion of Results

Teachers with widely-varying views on writing in mathematics incorporated writing activities into their classrooms and found it a useful instructional strategy. All participants judged the information gained from writing to be valuable. All intended to use writing activities in future mathematics courses. But no two viewed writing in exactly the same way.

Part way through the study, Mary said that writing was "a window into students' minds". This metaphor of 'writing as a window' pervaded teachers' comments. Ricoeur (1977)

views metaphors as disclosing, rather than disguising meaning. Thus, participants' metaphors disclosed the meanings writing had for them as mathematics teachers.

Windows can be used for observation only. Teachers spoke of getting "insight" into students attitudes and understanding , of "peeking inside students' heads", of "seeing" students feelings, attitudes and understandings. The term "insight" implies looking in from the outside with no actual contact, as one might look through the window of an operating theatre. One can see but not actually come into contact with what is on the other side of the window. By getting "insight", one can observe and evaluate. The window can be simultaneously a way of seeing and a barrier against interaction between inside and outside. There can be a distancing, a separation between the observer and the observed.

Windows, however, can also be opened to sunshine and gentle breezes, or to torrents of rain and gusting wind. Teachers also saw writing as allowing students to "release their feelings" and to "vent their anger". They were clearly aware of the observed coming into the side of the observer through the window of writing. The world on the other side of the window is not passively observed, but actively felt. The separation between observer and observed is gone.

A third image comes to mind. Hearing the voices of friends at play outside, a child opens the window to climb out and join them. One can use a window as a means of getting into an "other's" world. Teachers spoke of "getting into students' heads" of "getting into students' minds better". A window can be an avenue for active exploration. One can choose not just to see or to feel the world outside, but to go into it, to follow the voices.

Just as a window can be used in many ways, so too can writing activities in mathematics. The teacher can choose to observe the student from a distance. He or she can choose to feel the world of the student. Or the teacher can enter the student's world by following his or her voice as expressed in writing.

Participants in this study envisaged one or more of these images of writing in mathematics. Through metaphor, teachers constructed and expressed their understandings. Different teachers came to different understandings which were reflected in the ways in which they implemented programs of writing in their mathematics classrooms.

One cannot, therefore, speak of writing in mathematics with only one image in mind. Each teacher's use of writing activities was unique and varied over time as the particular situation, in which writing was used, varied.

Many factors affected these situations. Teachers' views of mathematics and teaching had a major effect on the ways in which writing activities were used. In particular, teachers' views of the roles of the student and the teacher determined the perspective from which writing was viewed. Related to this was the type of assessment teachers used. The externally-set examination containing written-response questions had a major impact on the way teachers viewed and used writing activities. Some mathematical topics seemed to lend themselves better to different types of writing activities. Some groups of students seemed to encourage teachers to seek information from them. Not least, the teacher's personality and style of teaching affected how writing was viewed. Even events in the teacher's personal life made a difference to the time and energy he or she had to devote to teaching.

Overall, the use of an instructional strategy like writing activities cannot be considered in isolation. It must be related to the context in which it is placed. The actors in the situation, teachers and students, are most influential.

Successes with particular instructional techniques depend on the teacher and the classroom context (Baird & Mitchell, 1986). While curricular changes and the use of new teaching techniques can be dictated, the curriculum in

practice is filtered through the attitudes, conceptions and behaviours of teachers.

> Teachers teach an externally mandated curriculum, but behind the classroom doors they interpret and modify the curriculum in a manner that reflects the personality, the educational philosophy and style of the teacher as well as the voices, the needs, the activities, and the influences of the students. (Van Manen, 1992, p.14)

As programs of writing activities vary with the context in which they are used, so does the awareness that each teacher gains and comments on. "Everything is said by an observer" (Maturana, 1987, p. 65).

Many understandings were contained in students' responses. Some were considered and accepted; others were rejected. Several factors assisted in determining how teachers reacted. The teacher's background knowledge in such areas as usage of mathematical language and conceptions of mathematics influenced their awareness and subsequent interpretation of student responses. Teachers' views of the roles of the student and the teacher affected the ways teachers reacted to suggestions and criticism from students, as did the teacher's overall confidence.

Teachers' emotions also played a part. When students responded angrily, appearing to reject all that the teacher stood for, it was natural for the teacher to feel rejected too. Teachers' previously-acquired knowledge of students also affected the light in which their responses were viewed.

In spite of these influences, teachers did use student responses to evaluate their mathematics instruction. Being sensitive to criticism from others is human nature. It may be a reflection of the quality of the participants, that in spite of this very natural tendency, they often did hear what students had to say and did make changes in response.

Participants reflected on their conceptions of mathematics and teaching. It is not clear whether teachers were aware of reflecting. A number of factors may have caused them to do so. The introduction of any new teaching technique causes teachers to examine how it can best add to their instruction of students. Any change creates the need for evaluation of what has been done previously.

Weekly interviews with the researcher may have contributed. Discussion causes interaction of participants' ideas. The need to articulate forces clarification of thoughts and a partial detachment of ideas from the speaker (Skemp, 1987). These factors lead naturally to reflection.

Writing itself may possess unique attributes that foster self-examination.

> Major reforms in instruction may carry with them new definitions of what it means to teach and learn. If these reforms are adopted fully, they will lead to fundamental changes in teachers' notions of teaching and learning in their subject areas. (Langer & Applebee, 1987, p. 73)

This study provides evidence to support the view that writing is such a major reform in mathematics teaching. Accepting that students have a voice in education must have an impact on teachers' views of learning and teaching.

While some factors enhanced reflection, others hindered the actions that teachers took in response. Teachers did not have time to reteach lessons or provide individual remediation. All participants commented on the pressure they felt to complete the course on time. Implementation of a new program always necessitates more work for teachers. Becoming familiar with new instructional techniques and preparing new teaching materials and tests are time-consuming activities on top of what teachers felt were already heavy loads. These new demands left little time for teachers to consider modifying their attitudes or instruction based on students' feedback. That participants made as many changes as they did reflects their commitment to teaching and students and their view that feedback from students is important.

Teachers lacked professional guidance and support for implementing changes based on the information contained in student writing. Such help would be of great value in suggesting ways to help students to change their use of mathematics language, their conceptions of mathematics, or their modes of understanding.

Assistance for teachers is needed in the form of relevant professional development, time provided for meetings with colleagues where problems can be discussed and plans be made, recognition of the very real problems teachers face in the classroom, and support for the attempts teachers make to deal with these problems under the many constraints placed on them. Schools at present tend to prevent these types of assistance by withholding time, encouragement and responsibility from teachers who try to work cooperatively (Johnson, 1990; Lortie, 1975).

Teachers took students' messages personally. When students broke promises, teachers felt let down. When students raged at mathematics, teachers felt injured. In one respect, this is good, for it means that teachers are sensitive to their students' feelings and attitudes. It would, however, seem to be a major obstacle to teachers continuing to seek out this kind of feedback.

Teachers need assistance in understanding why students come to have such negative feelings and attitudes. They must be able to discuss their problems and perceptions of students with colleagues. Help in finding out ways of dealing with these students' feelings, and support and encouragement while trying to do so are also required. Teachers are not likely, otherwise, to endure the 'darkness' contained in some student writing for the sake of a few 'bright spots'.

Suggestions for Further Research

The teachers involved in this study varied in some respects. Teachers of different genders, training, and experience participated. However, all taught academic students in their final year of high school. This study could be replicated to determine whether teachers would gain the same awareness and reflection if writing activities were used at different grade levels or with non-academic students. Male and female students or students with different cultural backgrounds may relate different information in writing.

Another fertile area of research is the effect of expressive writing on students. When students wrote to record their feelings for the teacher in transactional writing, no changes in work habits resulted. However, when students wrote to explore their own feelings in expressive writing, attitudes did change. It would appear that teachers' and students' views of writing are important. These determine both the mode of writing and the amount of reflection done by students when writing. Expressive writing may play an important part in changing students' attitudes to studying mathematics.

The teacher's view of writing in mathematics affects the ways in which he or she uses it in the classroom. As a child's teachers change from year to year, so too would his

or her experiences and perspectives on writing. An external examination may impose yet another view of writing on students. Investigation is needed into the effects of exposure to potentially frequent changes in views on writing.

While changes in teachers' practices, attitudes and conceptions were noted in this study, the time period involved was very short. A study examining the use of writing activities by mathematics teachers over a longer period might find more noticeable or different results.

This study indicated that teachers may change their conceptions of writing in mathematics and, thus, their conceptions of mathematics and mathematics teaching. Continuing discussion between researchers and teachers would seem to be an important factor in these changes. Other research studies have found that ongoing inservice support during periods of curricular change is more effective than support given just at the beginning of such a period (Baird & Mitchell, 1986; Wood, Cobb & Yackel, 1991). This may have implications for teacher education and teacher professional development in general. Studies examining the role of curricular support while implementing curricular change would add much to the area.

> It may be much more important to give teachers new frameworks for understanding what to count as learning than it is to give them new activities or curricula. (Langer & Applebee, p. 87)

Long-term studies might be carried out to see what happens when such support is withdrawn.

The teacher's conception of writing in mathematics is clearly related to his or her philosophy of mathematics. "Whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (Thom, 1973, p. 204). Teachers' own conceptions of mathematics affected what they saw in student responses. They recognized that their views might influence students and that students' views, in turn, might affect theirs. The interactions between teachers' and students' conceptions of mathematics certainly have an effect on both teaching and learning. Writing is only one way in which these conceptions interact. This area is a fruitful one for future research.

This study focused on what happens to teachers when students are given a voice in the mathematics classroom through writing. It is also important to explore what happens to students when they have a chance to speak.

Restricting informal language use in mathematics effectively prevents many students from participating fully in the mathematics community. Bernstein (1971) contrasts elaborated codes, where speech tends to be explicit and independent of context, with restricted codes, which are context dependent. Meaning tends to be conveyed implicitly and is available only to a particular audience, which

understands unspoken, but relevant features.

The use of the very restricted code of formal mathematical language restricts access to the mathematics behind it to those who understand its unspoken, but relevant features. When people talk in "mathematics", they cut away any sensory references that might distract one from the central concept or introduce emotions or values.

> But the act of cutting out much of what the natural language can and must say also rearranges the world. Natural language represents the blend of naming and valuing that is characteristically human. (Lloyd-Jones, 1982, p. 121)

Abstraction in mathematics, thus, causes alienation.

Writing about mathematics in informal everyday language has the effect of making the student and the teacher use elaborated codes, thus increasing access to mathematical understandings. The resulting 'de-mystification' of mathematics might revolutionize both mathematics and mathematics teaching (Hersch, 1986). If the notion of encouraging students to communicate mathematics gained ground, then more people might find the study of mathematics both engaging and rewarding (Pimm, 1987). Research needs to be carried out investigating ways in which students can express mathematics for themselves and the effects of this expression on students and teachers alike.

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